Formularium tweedegraadsfuncties

Definitie

$f(x) = ax^2 + bx + c$, *met* $a, b, c \in \mathbb{R}$ *en* $a \neq 0$.

Bespreking van de grafiek

Symmetrieas: *de rechte met vergelijking* $x = -\dfrac{b}{2a}$

Top: $T\left(-\dfrac{b}{2a}, \dfrac{-b^2+4ac}{4a}\right)$, *of nog* $T\left(-\dfrac{b}{2a}, \dfrac{-\Delta}{4a}\right)$

Nulpunten: *Stel* $\Delta = b^2 - 4ac$

1) $\Delta < 0$*: geen (reële) nulpunten*

2) $\Delta = 0$*: één nulpunt (of twee samenvallende nulpunten)* $\rightarrow \left(-\dfrac{b}{2a}, 0\right)$

3) $\Delta > 0$*: twee verschillende nulpunten* $\rightarrow \left(-\dfrac{b+\sqrt{\Delta}}{2a}, 0\right)$ *en* $\left(-\dfrac{b-\sqrt{\Delta}}{2a}, 0\right)$

(De nulwaarden worden ook vaak met x_1 *en* x_2 *genoteerd, waarbij geldt* $x_1 < x_2$ *)*

Vorm: *De grafiek van een tweedegraadsfunctie is een parabool*

Als $a > 0$ *dan spreken we van een dalparabool*

Als $a < 0$ *dan spreken we van een bergparabool*

Eigenschappen van de wortels

Voor de wortels x_1 *en* x_2 *van de vierkantsvergelijking* $ax^2 + bx + c = 0$ *geldt:*

$$S = x_1 + x_2 = \frac{-b}{a} \quad en \quad P = x_1 . x_2 = \frac{c}{a}$$

Ontbinden in factoren

Als $\Delta = b^2 - 4ac > 0$ *dan geldt:* $ax^2 + bx + c = a(x - x_1)(x - x_2)$

Als $\Delta = b^2 - 4ac = 0$ *dan geldt:* $ax^2 + bx + c = a\left(x + \dfrac{b}{2a}\right)^2$

Als $\Delta = b^2 - 4ac < 0$ *dan kan* $ax^2 + bx + c$ *niet verder ontbonden worden in factoren*

Tekenverloop

<table>
<tr><td></td><td>$a > 0$</td><td>$a < 0$</td></tr>
<tr><td>$\Delta < 0$</td><td>
<table>
<tr><td>x</td><td>$-\infty$</td><td></td><td>$+\infty$</td></tr>
<tr><td>$f(x)$</td><td>+</td><td>+</td><td>+</td></tr>
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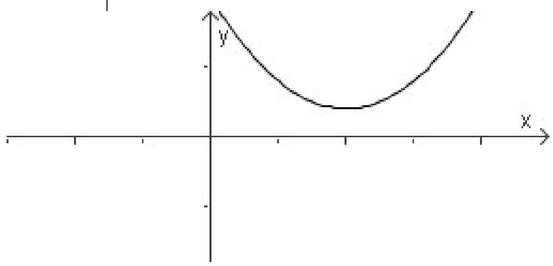

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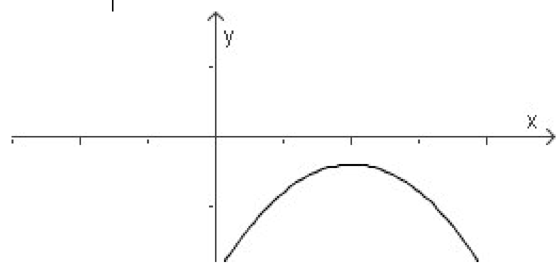

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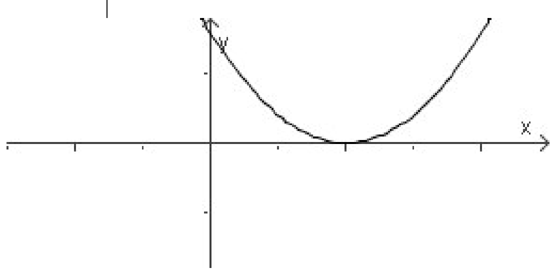

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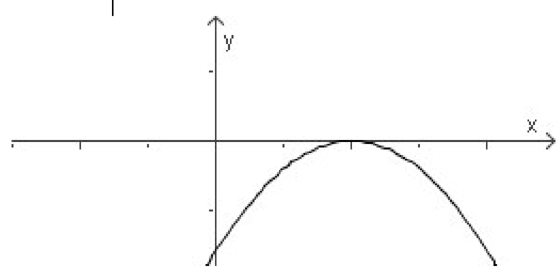

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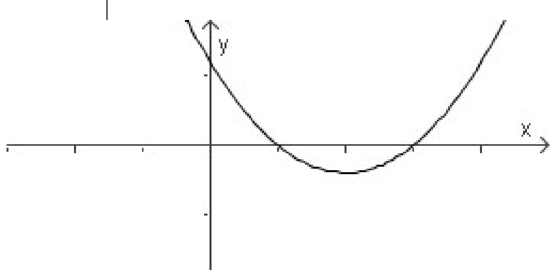

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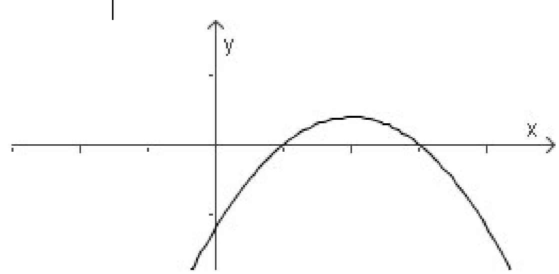

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