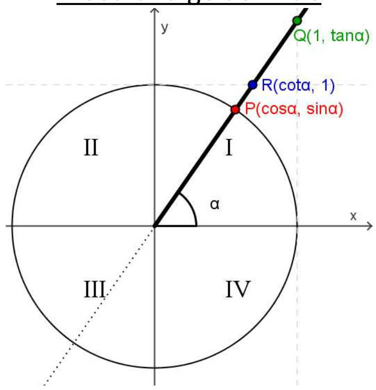
Formularium goniometrie

Meetkundige definitie



Definities

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$$

$$\cot\alpha = \frac{\cos\alpha}{\sin\alpha}$$

$$\sec\alpha = \frac{1}{\cos\alpha}$$

$$\csc\alpha = \frac{1}{\sin\alpha}$$

Hoofdformules

$$\sin^2\alpha + \cos^2\alpha = 1$$

$$\tan^2\alpha + 1 = \sec^2\alpha = \frac{1}{\cos^2\alpha}$$

$$\cot^2\alpha + 1 = \csc^2\alpha = \frac{1}{\sin^2\alpha}$$

$$\cot\alpha = \frac{1}{\tan\alpha}$$

Bekende hoeken

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin\alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan\alpha$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$\notin\mathbb{R}$
$\cot\alpha$	$\notin\mathbb{R}$	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0

Verwante hoeken

Tegengestelde hoeken	*Supplementaire hoeken*
$\sin(-\alpha) = -\sin\alpha$	$\sin(\pi-\alpha) = \sin\alpha$
$\cos(-\alpha) = \cos\alpha$	$\cos(\pi-\alpha) = -\cos\alpha$
$\tan(-\alpha) = -\tan\alpha$	$\tan(\pi-\alpha) = -\tan\alpha$
$\cot(-\alpha) = -\cot\alpha$	$\cot(\pi-\alpha) = -\cot\alpha$
Complementaire hoeken	***Anti-supplementaire hoeken***
$\sin(\pi/2-\alpha) = \cos\alpha$	$\sin(\pi+\alpha) = -\sin\alpha$
$\cos(\pi/2-\alpha) = \sin\alpha$	$\cos(\pi+\alpha) = -\cos\alpha$
$\tan(\pi/2-\alpha) = \cot\alpha$	$\tan(\pi+\alpha) = \tan\alpha$
$\cot(\pi/2-\alpha) = \tan\alpha$	$\cot(\pi+\alpha) = \cot\alpha$

Som en verschil formules

$$\cos(\alpha-\beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos(\alpha+\beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$$

$$\sin(\alpha-\beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\sin(\alpha+\beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha.\tan\beta}$$

$$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha.\tan\beta}$$

Verdubbelingsformules

$$\sin 2\alpha = 2.\sin\alpha.\cos\alpha$$

$$\cos 2\alpha = 2\cos^2\alpha - 1$$

$$= 1 - 2\sin^2\alpha$$

$$= \cos^2\alpha - \sin^2\alpha$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$$

Halveringsformules (Carnot)

$$\cos^2\alpha = \frac{1+\cos 2\alpha}{2}$$

$$\sin^2\alpha = \frac{1-\cos 2\alpha}{2}$$

$$\tan^2\alpha = \frac{1-\cos 2\alpha}{1+\cos 2\alpha}$$

Formules van Simpson (product → som)

$$2\sin\alpha\cos\beta = \sin(\alpha-\beta) + \sin(\alpha+\beta)$$

$$2\cos\alpha\cos\beta = \cos(\alpha-\beta) + \cos(\alpha+\beta)$$

$$2\sin\alpha\sin\beta = \cos(\alpha-\beta) - \cos(\alpha+\beta)$$

Formules van Simpson (som → product)

$$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin x - \sin y = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

t-formules ($t = \tan\frac{x}{2}$)

$$\tan x = \frac{2t}{1-t^2} \qquad \sin x = \frac{2t}{1+t^2} \qquad \cos x = \frac{1-t^2}{1+t^2}$$