1. Los op (zonder GRM):

a) $\begin{cases} x-3y+7z=4 \\ 2x-7y+z=-7 \\ 5x+2y-10z=9 \end{cases} \Rightarrow A_b = \left[\begin{array}{ccc|c} 1 & -3 & 7 & 4 \\ 2 & -7 & 1 & -7 \\ 5 & 2 & -10 & 9 \end{array}\right] \overset{R_2-2R_1}{\underset{R_3-5R_1}{\sim}} \left[\begin{array}{ccc|c} 1 & -3 & 7 & 4 \\ 0 & -1 & -13 & -15 \\ 0 & 17 & -45 & -11 \end{array}\right]$

$\overset{R_1-3R_2}{\underset{R_3+17R_2}{\sim}} \left[\begin{array}{ccc|c} 1 & 0 & 46 & 49 \\ 0 & -1 & -13 & -15 \\ 0 & 0 & -266 & -266 \end{array}\right] \overset{\frac{R_2}{-1}}{\underset{\frac{R_3}{-266}}{\sim}} \left[\begin{array}{ccc|c} 1 & 0 & 46 & 49 \\ 0 & 1 & 13 & 15 \\ 0 & 0 & 1 & 1 \end{array}\right] \overset{R_1-46R_3}{\underset{R_2-13R_3}{\sim}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right] \Rightarrow V=\{(3,2,1)\}$

b) $\begin{cases} 4x+21y+29z=16 \\ 2x+24y+28z=17 \end{cases} \Rightarrow A_b = \left[\begin{array}{ccc|c} 4 & 21 & 29 & 16 \\ 2 & 24 & 28 & 17 \end{array}\right] \overset{2R_2-R_1}{\sim} \left[\begin{array}{ccc|c} 4 & 21 & 29 & 16 \\ 0 & 27 & 27 & 18 \end{array}\right] \overset{\frac{R_2}{9}}{\sim} \left[\begin{array}{ccc|c} 4 & 21 & 29 & 16 \\ 0 & 3 & 3 & 2 \end{array}\right]$

$\overset{R_1-7R_2}{\sim} \left[\begin{array}{ccc|c} 4 & 0 & 8 & 2 \\ 0 & 3 & 3 & 2 \end{array}\right] \overset{\frac{R_1}{4}}{\underset{\frac{R_2}{3}}{\sim}} \left[\begin{array}{ccc|c} 1 & 0 & 2 & 1/2 \\ 0 & 1 & 1 & 2/3 \end{array}\right] \Rightarrow V=\{(1/2-2k,\, 2/3-k, k) \parallel k\in\mathbb{R}\}$

c) $\begin{cases} 4x-3y+7z=4 \\ -2x-7y+z=-7 \\ 3x+2y+3z=1 \end{cases} \Rightarrow A_b = \left[\begin{array}{ccc|c} 4 & -3 & 7 & 4 \\ -2 & -7 & 1 & -7 \\ 3 & 2 & 3 & 1 \end{array}\right] \overset{2R_2+R_1}{\underset{4R_3-3R_1}{\sim}} \left[\begin{array}{ccc|c} 4 & -3 & 7 & 4 \\ 0 & -17 & 9 & -10 \\ 0 & 17 & -9 & -8 \end{array}\right]$

$\underset{R_3+R_2}{\sim} \left[\begin{array}{ccc|c} 4 & -3 & 7 & 4 \\ 0 & -17 & 9 & -10 \\ 0 & 0 & 0 & -18 \end{array}\right] \Rightarrow V=\varnothing$

2. Welk getal bestaande uit vier cijfers voldoet aan deze eigenschappen:
   - De som van de cijfers is 19.
   - De som van het getal en het getal dat we krijgen door de volgorde van de cijfers om te keren is 9218.
   - Het cijfer van de tientallen is een meer dan het cijfer van de duizendtallen.
   - Het cijfer van de honderdtallen is twee meer dan het cijfer van de eenheden.

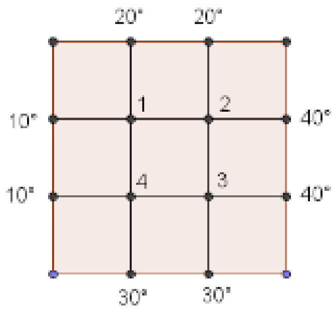
   Noem dat getal $(abcd)_{10}=1000a+100b+10c+d$, dan worden de gegevens:

   - $a+b+c+d=19$
   - $1000a+100b+10c+d+1000d+100c+10b+a=9218 \Leftrightarrow 1001a+110b+110c+1001d=9218$
   - $c=a+1$
   - $b=d+2$

$\begin{cases} a+b+c+d=19 \\ 91a+11b+11c+91d=838 \\ -a+c=1 \\ b-d=2 \end{cases} \Rightarrow A_b = \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 19 \\ 91 & 10 & 10 & 91 & 838 \\ -1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 & 2 \end{array}\right] \overset{rref}{\sim} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 8 \\ 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 1 & 1 & 9 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

Dus dit wordt: $\begin{cases} a=8-d \\ b=2+d \\ c=9-d \\ d\in\{1,2,3,4,5,6,7\} \end{cases}$, dus het getal kan zijn: 7381, 6472, 5563, 4654, 3745, 2836 of 1927.

3. Bij de studie van warmtetransport wenst men de temperatuurverdeling bij evenwicht te kennen van een dunnen plaat, wanneer de temperatuur op de rand van de plaat gekend is.



We zoeken T_1, T_2, T_3 en T_4, de temperaturen in de 4 inwendige knopen. We weten dat de temperatuur in een knooppunt bij benadering gelijk is aan het gemiddelde van de temperaturen in de nabijgelegen knooppunten. Zo is voor de figuur hiernaast bijvoorbeeld $T_1 = \dfrac{10+20+T_2+T_4}{4}$.

Stel het stelsel op waarmee je de temperaturen T_1, T_2, T_3 en T_4 kan bepalen en los het op met je rekenmachine.

De gegevens worden: $$\begin{cases} T_1 = \dfrac{10+20+T_2+T_4}{4} \\ T_2 = \dfrac{T_1+20+40+T_3}{4} \\ T_3 = \dfrac{T_4+T_2+40+30}{4} \\ T_4 = \dfrac{10+T_1+T_3+30}{4} \end{cases} \Leftrightarrow \begin{cases} 4T_1 - T_2 - T_4 = 30 \\ -T_1 + 4T_2 - T_3 = 60 \\ -T_2 + 4T_3 - T_4 = 70 \\ -T_1 - T_3 + 4T_4 = 40 \end{cases} \overset{rref}{\Leftrightarrow} \begin{cases} T_1 = 20 \\ T_2 = 27{,}5 \\ T_3 = 30 \\ T_4 = 22{,}5 \end{cases}.$$

4. Bespreek de oplossingenverzameling van volgende stelsels in functie van parameter $\lambda \in \mathbb{R}$.

a) $$\begin{cases} \lambda x_1 + x_2 + x_3 = 2 \\ x_1 + \lambda x_2 + x_3 = 3 \end{cases} \Rightarrow A_b = \left[\begin{array}{ccc|c} 1 & \lambda & 1 & 3 \\ \lambda & 1 & 1 & 2 \end{array}\right] \overset{R_2-\lambda R_1}{\sim} \left[\begin{array}{ccc|c} 1 & \lambda & 1 & 3 \\ 0 & 1-\lambda^2 & 1-\lambda & 2-3\lambda \end{array}\right]$$

$$\boxed{\lambda \neq 1 \wedge \lambda \neq -1} : A_b \underset{\frac{R_2}{1-\lambda^2}}{\sim} \left[\begin{array}{ccc|c} 1 & \lambda & 1 & 3 \\ 0 & 1 & \dfrac{1}{1+\lambda} & \dfrac{2-3\lambda}{1-\lambda^2} \end{array}\right] \underset{R_1-\lambda R_2}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & \dfrac{1}{1+\lambda} & \dfrac{3-2\lambda}{1-\lambda^2} \\ 0 & 1 & \dfrac{1}{1+\lambda} & \dfrac{2-3\lambda}{1-\lambda^2} \end{array}\right]$$

Dan is $V = \left\{ \left(\dfrac{3-2\lambda}{1-\lambda^2} - \dfrac{k}{1+\lambda}, \dfrac{2-3\lambda}{1-\lambda^2} - \dfrac{k}{1+\lambda}, k \right) \| \, k \in \mathbb{R} \right\}$

$\boxed{\lambda = 1} : A_b \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 0 & 0 & -1 \end{array}\right]$. Dan is $V = \varnothing$.

$\boxed{\lambda = -1} : A_b \sim \left[\begin{array}{ccc|c} 1 & -1 & 1 & 3 \\ 0 & 0 & 2 & 5 \end{array}\right] \underset{\frac{R_2}{2}}{\overset{R_1-\frac{R_2}{2}}{\sim}} \left[\begin{array}{ccc|c} 1 & -1 & 0 & 1/2 \\ 0 & 0 & 1 & 5/2 \end{array}\right]$. Dan is $V = \left\{ (1/2 + k, k, 5/2) \, \| \, k \in \mathbb{R} \right\}$.

b) $$\begin{cases} x_1 - 2x_3 = \lambda + 4 \\ -2x_1 + \lambda x_2 + 7x_3 = -14 \\ -x_1 + \lambda x_2 + 6x_3 = \lambda - 12 \end{cases} \Rightarrow A_b = \left[\begin{array}{ccc|c} 1 & 0 & -2 & \lambda+4 \\ -2 & \lambda & 7 & -14 \\ -1 & \lambda & 6 & \lambda-12 \end{array}\right] \underset{R_3+R_1}{\overset{R_2+2R_1}{\sim}} \left[\begin{array}{ccc|c} 1 & 0 & -2 & \lambda+4 \\ 0 & \lambda & 3 & 2\lambda-6 \\ 0 & \lambda & 4 & 2\lambda-8 \end{array}\right]$$

$$\overset{R_3-R_2}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & -2 & \lambda+4 \\ 0 & \lambda & 3 & 2\lambda-6 \\ 0 & 0 & 1 & -2 \end{array}\right] \underset{R_2-3R_3}{\overset{R_1+2R_3}{\sim}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & \lambda \\ 0 & \lambda & 0 & 2\lambda \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$$\boxed{\lambda \neq 0}: A_b = \left[\begin{array}{ccc|c} 1 & 0 & 0 & \lambda \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -2 \end{array}\right]. \text{ Dan is } V = \{(\lambda, 2, -2)\}.$$

$$\boxed{\lambda = 0}: A_b = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 \end{array}\right]. \text{ Dan is } V = \{(0, k, -2) \,\|\, k \in \mathbb{R}\}.$$

5. Bespreek de oplossingenverzameling van $\begin{cases} ax + bz = 2 \\ ax + ay + 4z = 4 \\ ay + 2z = b \end{cases}$ in functie van de parameters $a, b \in \mathbb{R}$.

$$A_b = \left[\begin{array}{ccc|c} a & 0 & b & 2 \\ a & a & 4 & 4 \\ 0 & a & 2 & b \end{array}\right] \overset{R_2 - R_1}{\sim} \left[\begin{array}{ccc|c} a & 0 & b & 2 \\ 0 & a & 4-b & 2 \\ 0 & a & 2 & b \end{array}\right] \overset{R_3 - R_2}{\sim} \left[\begin{array}{ccc|c} a & 0 & b & 2 \\ 0 & a & 4-b & 2 \\ 0 & 0 & b-2 & b-2 \end{array}\right]$$

$$\boxed{b \neq 2}: A_b \overset{\frac{R_3}{b-2}}{\sim} \left[\begin{array}{ccc|c} a & 0 & b & 2 \\ 0 & a & 4-b & 2 \\ 0 & 0 & 1 & 1 \end{array}\right] \underset{R_2 + (b-4)R_3}{\overset{R_1 - bR_3}{\sim}} \left[\begin{array}{ccc|c} a & 0 & 0 & 2-b \\ 0 & a & 0 & b-2 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$$\boxed{a \neq 0}: A_b \underset{\frac{R_2}{a}}{\overset{\frac{R_1}{a}}{\sim}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{2-b}{a} \\ 0 & 1 & 0 & \frac{b-2}{a} \\ 0 & 0 & 1 & 1 \end{array}\right] \Rightarrow als\ b \neq 2 \wedge a \neq 0 : V = \left\{\left(\frac{2-b}{a}, \frac{b-2}{a}, 1\right)\right\}.$$

$$\boxed{a = 0}: A_b = \left[\begin{array}{ccc|c} 0 & 0 & 0 & 2-b \\ 0 & 0 & 0 & b-2 \\ 0 & 0 & 1 & 1 \end{array}\right] \Rightarrow als\ b \neq 2 \wedge a = 0 : V = \varnothing.$$

$$\boxed{b = 2}: A_b \sim \left[\begin{array}{ccc|c} a & 0 & 2 & 2 \\ 0 & a & 2 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\boxed{a \neq 0}: A_b \underset{\frac{R_2}{a}}{\overset{\frac{R_1}{a}}{\sim}} \left[\begin{array}{ccc|c} 1 & 0 & 2/a & 2/a \\ 0 & 1 & 2/a & 2/a \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow als\ b = 2 \wedge a \neq 0 : V = \left\{\left(\frac{2-2k}{a}, \frac{2-2k}{a}, k\right) \,\|\, k \in \mathbb{R}\right\}.$$

$$\boxed{a = 0}: A_b \underset{\frac{R_2}{2}}{\overset{\frac{R_1}{2}}{\sim}} \left[\begin{array}{ccc|c} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow als\ b = 2 \wedge a = 0 : V = \{(k, m, 1) \,\|\, k, m \in \mathbb{R}\}.$$