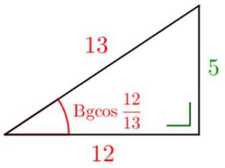
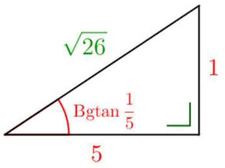
1. Bereken “zonder” je rekenmachine:

a) $$\sin\left(\text{Bgcos}\,\frac{12}{13}\right)=\sqrt{1-\left(\frac{12}{13}\right)^2}=\sqrt{\frac{25}{169}}=\frac{5}{13}$$



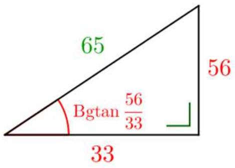
b) $$\cos\left(2\,\text{Bgtan}\,\frac{1}{5}\right)=2\cos^2\left(\text{Bgtan}\,\frac{1}{5}\right)-1=2.\left(\frac{5}{\sqrt{26}}\right)^2-1=\frac{12}{13}$$

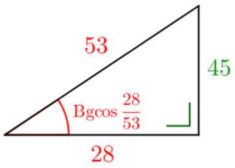


c) $$\sin\left(\text{Bgcos}\,\frac{28}{53}+\text{Bgtan}\,\frac{56}{33}\right)$$

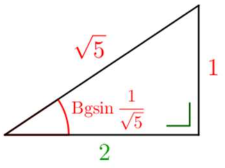
$$=\sin\left(\text{Bgcos}\,\frac{28}{53}\right)\cos\left(\text{Bgtan}\,\frac{56}{33}\right)+\cos\left(\text{Bgcos}\,\frac{28}{53}\right)\sin\left(\text{Bgtan}\,\frac{56}{33}\right)$$

$$=\frac{45}{53}\cdot\frac{33}{65}+\frac{28}{53}\cdot\frac{56}{65}=\frac{3053}{3445}$$





d) $$\tan\left(\text{Bgsin}\,\frac{1}{\sqrt{5}}-2\text{Bgtan}\,(-3)\right)=\frac{\tan\left(\text{Bgsin}\,\frac{1}{\sqrt{5}}\right)-\tan\left(2\text{Bgtan}\,(-3)\right)}{1+\tan\left(\text{Bgsin}\,\frac{1}{\sqrt{5}}\right)\cdot\tan\left(2\text{Bgtan}\,(-3)\right)}$$



$\overset{**}{=}\dfrac{\frac{1}{2}-\frac{3}{4}}{1+\frac{1}{2}\cdot\frac{3}{4}}=-\dfrac{2}{11}$ (want: $**:\tan\left(2\text{Bgtan}\,(-3)\right)=\dfrac{2\tan\left(\text{Bgtan}\,(-3)\right)}{1-\tan^2\left(\text{Bgtan}\,(-3)\right)}=\dfrac{-6}{1-9}=\dfrac{3}{4}$)

2. Bereken zonder je rekenmachine:

a) $\text{Bgsin}\,\dfrac{\sqrt{2}}{2}=\dfrac{\pi}{4}$ (want $\sin\dfrac{\pi}{4}=\dfrac{\sqrt{2}}{2}$ en $\dfrac{\pi}{4}\in\left[-\dfrac{\pi}{2},\dfrac{\pi}{2}\right]$)

b) $\text{Bgtan}\left(-\dfrac{\sqrt{3}}{3}\right)=-\dfrac{\pi}{6}$ (want $\tan\dfrac{-\pi}{6}=\dfrac{-\sqrt{3}}{3}$ en $\dfrac{-\pi}{6}\in\left[-\dfrac{\pi}{2},\dfrac{\pi}{2}\right]$)

c) $\text{Bgcos}\left(\cos\left(\dfrac{7\pi}{6}\right)\right)=\dfrac{5\pi}{6}$ (want $\cos\dfrac{7\pi}{6}\overset{TH}{=}\cos\dfrac{5\pi}{6}$ en $\dfrac{5\pi}{6}\in[0,\pi]$)

d) $\text{Bgtan}\,\dfrac{1}{4}+\text{Bgtan}\,\dfrac{3}{5}=\dfrac{\pi}{4}$

$$\text{Bgtan}\,\frac{1}{4}+\text{Bgtan}\,\frac{3}{5}=x\overset{!!}{\Rightarrow}\tan\left(\text{Bgtan}\,\frac{1}{4}+\text{Bgtan}\,\frac{3}{5}\right)=\tan x\Leftrightarrow\frac{\frac{1}{4}+\frac{3}{5}}{1-\frac{1}{4}\cdot\frac{3}{5}}=\tan x\Leftrightarrow\tan x=1$$

We hebben dus bewezen dat er een $k\in\mathbb{Z}$ is zodat $\text{Bgtan}\,\dfrac{1}{4}+\text{Bgtan}\,\dfrac{3}{5}=\dfrac{\pi}{4}+k\pi$.

Maar omdat $0<\text{Bgtan}\,\dfrac{1}{4}<\text{Bgtan}\,\dfrac{3}{5}<\dfrac{\pi}{2}$, geldt dat $0<\text{Bgtan}\,\dfrac{1}{4}+\text{Bgtan}\,\dfrac{3}{5}<\pi$ zodat de juiste waarde $k=0$ is.

e) $2\ \text{Bgtan}\ 5-\text{Bgtan}\ \frac{7}{17}=\frac{3\pi}{4}$ (merk op dat $\tan\left(2\ \text{Bgtan}\ 5\right)=\frac{2\tan\left(\text{Bgtan}\ 5\right)}{1-\tan^2\left(\text{Bgtan}\ 5\right)}=\frac{2.5}{1-25}=\frac{-5}{12}$)

$$2\ \text{Bgtan}\ 5-\text{Bgtan}\ \frac{7}{17}=x \overset{!!}{\Rightarrow} \tan x=\tan\left(2\ \text{Bgtan}\ 5-\text{Bgtan}\ \frac{7}{17}\right)=\frac{-\frac{5}{12}-\frac{7}{17}}{1-\frac{5}{12}\cdot\frac{7}{17}}=-1.$$

We hebben dus bewezen dat er een $k\in\mathbb{Z}$ is zodat $2\ \text{Bgtan}\ 5-\text{Bgtan}\ \frac{7}{17}=-\frac{\pi}{4}+k\pi$.

Maar omdat $\frac{\pi}{4}<\text{Bgtan}\ 5<\frac{\pi}{2}$ en $0<\text{Bgtan}\ \frac{7}{17}<\frac{\pi}{4}$ zal $\frac{\pi}{4}<2\ \text{Bgtan}\ 5-\text{Bgtan}\ \frac{7}{17}<\pi$ zodat de juiste waarde $k=1$ is.

f) $\text{Bgsin}\ \frac{1}{\sqrt{5}}+\text{Bgtan}\ \frac{1}{3}=\frac{\pi}{4}$ (we bewezen in oefening 1d al dat $\tan\left(\text{Bgsin}\ \frac{1}{\sqrt{5}}\right)=\frac{1}{2}$)

$$\text{Bgsin}\ \frac{1}{\sqrt{5}}+\text{Bgtan}\ \frac{1}{3}=x \overset{!!}{\Rightarrow} \tan x=\tan\left(\text{Bgsin}\ \frac{1}{\sqrt{5}}+\text{Bgtan}\ \frac{1}{3}\right)=\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\cdot\frac{1}{3}}=1$$

We hebben dus bewezen dat er een $k\in\mathbb{Z}$ is zodat $\text{Bgsin}\ \frac{1}{\sqrt{5}}+\text{Bgtan}\ \frac{1}{3}=\frac{\pi}{4}+k\pi$.

Maar omdat $0<\text{Bgsin}\ \frac{1}{\sqrt{5}}<\frac{\pi}{2}$ en $0<\text{Bgtan}\ \frac{1}{3}<\frac{\pi}{2}$, geldt dat $0<\text{Bgsin}\ \frac{1}{\sqrt{5}}+\text{Bgtan}\ \frac{1}{3}<\pi$ zodat de juiste waarde $k=0$ is.

3. Los de volgende vergelijking op: $\text{Bgsin}\ x+\text{Bgsin}\ \frac{8}{17}=\frac{\pi}{6}$

$$\overset{!!}{\Rightarrow} \sin\left(\text{Bgsin}\ x+\text{Bgsin}\ \frac{8}{17}\right)=\sin\frac{\pi}{6} \Leftrightarrow x.\cos\text{Bgsin}\ \frac{8}{17}+\cos\left(\text{Bgsin}\ x\right).\frac{8}{17}=\frac{1}{2}$$

$$\Leftrightarrow x\cdot\frac{15}{17}+\sqrt{1-x^2}\cdot\frac{8}{17}=\frac{1}{2} \Leftrightarrow 16.\sqrt{1-x^2}=17-30x \overset{\boxed{KV}}{\Rightarrow} 256\left(1-x^2\right)=289-1020x+900x^2$$

$$\Leftrightarrow 1156x^2-1020x+33=0 \Leftrightarrow \xcancel{x=\frac{15+8\sqrt{3}}{34}} \vee x=\frac{15-8\sqrt{3}}{34} \qquad \boxed{Stel\ x_1=\frac{15-8\sqrt{3}}{34}}$$

Er is dus een $k\in\mathbb{Z}$ zodat $\text{Bgsin}\ x_1+\text{Bgsin}\ \frac{8}{17}=\frac{\pi}{6}+2k\pi \vee \text{Bgsin}\ x_1+\text{Bgsin}\ \frac{8}{17}=\frac{5\pi}{6}+2k\pi$.

De enige oplossing die werkt is de eerste vergelijking met $k=0$, want uit $0<\text{Bgsin}\ x_1<\frac{\pi}{6}$ en $0<\text{Bgsin}\ \frac{8}{17}<\frac{\pi}{6}$ volgt dat $0<\text{Bgsin}\ x_1+\text{Bgsin}\ \frac{8}{17}<\frac{\pi}{3}$.

Alternatieve methode: Uit de opgave volgt dat $\text{Bgsin}\ x=\frac{\pi}{6}-\text{Bgsin}\ \frac{8}{17}$

$\overset{!!}{\Rightarrow} x=\sin\left(\frac{\pi}{6}-\text{Bgsin}\ \frac{8}{17}\right)=\frac{1}{2}\cdot\cos\left(\text{Bgsin}\ \frac{8}{17}\right)-\frac{\sqrt{3}}{2}\cdot\frac{8}{17}=\frac{15-8\sqrt{3}}{34}$. De rest verloopt analoog.