1. Bewijs de formules:

a) $\sin(\alpha+\beta+\gamma)=\sin\alpha\cos\beta\cos\gamma+\cos\alpha\sin\beta\cos\gamma+\cos\alpha\cos\beta\sin\gamma-\sin\alpha\sin\beta\sin\gamma$

$$\begin{aligned}\sin(\alpha+\beta+\gamma)&=\sin(\alpha+\beta)\cos\gamma+\cos(\alpha+\beta)\sin\gamma\\&=(\sin\alpha\cos\beta+\cos\alpha\sin\beta)\cos\gamma+(\cos\alpha\cos\beta-\sin\alpha\sin\beta)\sin\gamma\\&=\sin\alpha\cos\beta\cos\gamma+\cos\alpha\sin\beta\cos\gamma+\cos\alpha\cos\beta\sin\gamma-\sin\alpha\sin\beta\sin\gamma\end{aligned}$$

b) $\cos(\alpha+\beta+\gamma)=\cos\alpha\cos\beta\cos\gamma-\sin\alpha\sin\beta\cos\gamma-\sin\alpha\cos\beta\sin\gamma-\cos\alpha\sin\beta\sin\gamma$

$$\begin{aligned}\cos(\alpha+\beta+\gamma)&=\cos(\alpha+\beta)\cos\gamma-\sin(\alpha+\beta)\sin\gamma\\&=(\cos\alpha\cos\beta-\sin\alpha\sin\beta)\cos\gamma-(\sin\alpha\cos\beta+\cos\alpha\sin\beta)\sin\gamma\\&=\cos\alpha\cos\beta\cos\gamma-\sin\alpha\sin\beta\cos\gamma-\sin\alpha\cos\beta\sin\gamma-\cos\alpha\sin\beta\sin\gamma\end{aligned}$$

c) $\tan(\alpha+\beta+\gamma)=\dfrac{\tan\alpha+\tan\beta+\tan\gamma}{1-\tan\alpha\tan\beta-\tan\alpha\tan\gamma-\tan\beta\tan\gamma}$

$$\begin{aligned}\frac{\sin(\alpha+\beta+\gamma)}{\cos(\alpha+\beta+\gamma)}&=\frac{\sin\alpha\cos\beta\cos\gamma+\cos\alpha\sin\beta\cos\gamma+\cos\alpha\cos\beta\sin\gamma-\sin\alpha\sin\beta\sin\gamma}{\cos\alpha\cos\beta\cos\gamma-\sin\alpha\sin\beta\cos\gamma-\sin\alpha\cos\beta\sin\gamma-\cos\alpha\sin\beta\sin\gamma}\\&=\frac{\dfrac{\sin\alpha\cos\beta\cos\gamma+\cos\alpha\sin\beta\cos\gamma+\cos\alpha\cos\beta\sin\gamma-\sin\alpha\sin\beta\sin\gamma}{\cos\alpha\cos\beta\cos\gamma}}{\dfrac{\cos\alpha\cos\beta\cos\gamma-\sin\alpha\sin\beta\cos\gamma-\sin\alpha\cos\beta\sin\gamma-\cos\alpha\sin\beta\sin\gamma}{\cos\alpha\cos\beta\cos\gamma}}\\&=\frac{\tan\alpha+\tan\beta+\tan\gamma-\tan\alpha\tan\beta\tan\gamma}{1-\tan\alpha\tan\beta-\tan\alpha\tan\gamma-\tan\beta\tan\gamma}\end{aligned}$$

2. Bewijs dat $\tan 3x=\dfrac{3\tan x-\tan^3 x}{1-3\tan^2 x}$

$$\begin{aligned}\tan 3x&=\tan(2x+x)=\frac{\tan 2x+\tan x}{1-\tan 2x.\tan x}=\frac{\dfrac{2\tan x}{1-\tan^2 x}+\tan x}{1-\dfrac{2\tan x}{1-\tan^2 x}.\tan x}\\&=\frac{2\tan x+\tan x(1-\tan^2 x)}{1-\tan^2 x-2\tan^2 x}=\frac{3\tan x-\tan^3 x}{1-3\tan^2 x}\end{aligned}$$

3. Bewijs dat in ΔABC met hoeken α,β en γ geldt: $\dfrac{\cot\alpha+\cot\beta}{\tan\alpha+\tan\beta}+\dfrac{\cot\beta+\cot\gamma}{\tan\beta+\tan\gamma}+\dfrac{\cot\gamma+\cot\alpha}{\tan\gamma+\tan\alpha}=1$.

Merk op dat $\dfrac{\cot\alpha+\cot\beta}{\tan\alpha+\tan\beta}=\dfrac{\dfrac{1}{\tan\alpha}+\dfrac{1}{\tan\beta}}{\tan\alpha+\tan\beta}=\dfrac{\dfrac{\cancel{\tan\beta+\tan\alpha}}{\tan\alpha\tan\beta}}{\cancel{\tan\alpha+\tan\beta}}=\dfrac{1}{\tan\alpha\tan\beta}$.

We weten ook dat $\gamma=\pi-(\alpha+\beta)$, zodat $\tan\gamma=-\tan(\alpha+\beta)$. We vinden zo dat:

$$LL=\frac{1}{\tan\alpha\tan\beta}+\frac{1}{\tan\beta\tan\gamma}+\frac{1}{\tan\gamma\tan\alpha}=\frac{\tan\alpha+\tan\beta-\tan(\alpha+\beta)}{\tan\alpha\tan\beta(-\tan\gamma)}=\frac{\tan\alpha+\tan\beta-\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}}{-\tan\alpha\tan\beta\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}}$$

$$=\frac{(1-\tan\alpha\tan\beta)(\tan\alpha+\tan\beta)-(\tan\alpha+\tan\beta)}{-\tan\alpha\tan\beta(\tan\alpha+\tan\beta)}=\frac{-\tan\alpha\tan\beta(\tan\alpha+\tan\beta)}{-\tan\alpha\tan\beta(\tan\alpha+\tan\beta)}=RL$$

4. Bewijs de identiteiten:

a) $\cot\alpha - \cot\beta = \dfrac{\sin(\beta-\alpha)}{\sin\alpha\sin\beta}$

$$LL = \frac{\cos\alpha}{\sin\alpha} - \frac{\cos\beta}{\sin\beta} = \frac{\cos\alpha\sin\beta - \sin\alpha\cos\beta}{\sin\alpha\sin\beta} = \frac{\sin(\beta-\alpha)}{\sin\alpha\sin\beta} = RL$$

b) $\tan(\alpha-\beta)\tan(\alpha+\beta) = \dfrac{\tan^2\alpha - \tan^2\beta}{1-\tan^2\alpha\tan^2\beta}$

$$LL = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}\cdot\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta} = \frac{\tan^2\alpha-\tan^2\beta}{1-\tan^2\alpha\tan^2\beta} = RL$$

c) $\tan\left(\dfrac{\pi}{4}+\alpha\right) + \tan\left(\dfrac{\pi}{4}-\alpha\right) = \dfrac{2}{\cos 2\alpha}$

$$LL = \frac{\tan\frac{\pi}{4}+\tan\alpha}{1-\tan\frac{\pi}{4}\tan a} + \frac{\tan\frac{\pi}{4}-\tan a}{1+\tan\frac{\pi}{4}\tan\alpha} = \frac{1+\tan\alpha}{1-\tan a} + \frac{1-\tan a}{1+\tan\alpha} = \frac{(1+\tan\alpha)^2+(1-\tan a)^2}{(1-\tan a)(1+\tan\alpha)}$$

$$= \frac{1+\cancel{2\tan\alpha}+\tan^2\alpha+1-\cancel{2\tan\alpha}+\tan^2\alpha}{1-\tan^2\alpha} = \frac{2\sec^2\alpha}{1-\tan^2\alpha} = \frac{\frac{2}{\cos^2\alpha}}{1-\frac{\sin^2\alpha}{\cos^2\alpha}} = \frac{2}{\cos^2\alpha-\sin^2\alpha}$$

$$= \frac{2}{\cos 2\alpha} = RL$$

5. Bewijs dat, als $\alpha+\beta+\gamma+\varphi=\pi$, dan $\cos\alpha\cos\beta+\cos\gamma\cos\varphi = \sin\alpha\sin\beta+\sin\gamma\sin\varphi$.

Als $\alpha+\beta+\gamma+\varphi=\pi$, dan is ook $\alpha+\beta = \pi-(\gamma+\varphi)$

$$\cos\alpha\cos\beta+\cos\gamma\cos\varphi = \sin\alpha\sin\beta+\sin\gamma\sin\varphi$$
$$\Leftrightarrow \cos\alpha\cos\beta-\sin\alpha\sin\beta = \sin\gamma\sin\varphi-\cos\gamma\cos\varphi$$
$$\Leftrightarrow \cos(\alpha+\beta) = -\cos(\gamma+\varphi)$$
$$\Leftrightarrow \cos(\alpha+\beta) = \cos(\pi-(\gamma+\varphi))$$
$$\Leftrightarrow \cos(\alpha+\beta) = \cos(\alpha+\beta) \quad \square$$

6. Als gegeven is dat $\begin{cases}\sin\alpha+\cos\beta = x\\ \cos\alpha+\sin\beta = y\end{cases}$, druk dan $\sin(\alpha+\beta)$ uit in functie van x en y.

$$x^2+y^2 = (\sin\alpha+\cos\beta)^2+(\cos\alpha+\sin\beta)^2$$
$$= \sin^2\alpha+2\sin\alpha\cos\beta+\cos^2\beta+\cos^2\alpha+2\cos\alpha\sin\beta+\sin^2\beta$$
$$= 2+2(\sin\alpha\cos\beta+\cos\alpha\sin\beta) = 2+2\sin(\alpha+\beta)$$
$$\Leftrightarrow \sin(\alpha+\beta) = \frac{x^2+y^2}{2}-1$$

7. Bewijs, als in een driehoek met hoeken α,β,γ geldt dat $\cos 4\alpha+\cos 4\beta+\cos 4\gamma=-1$, dat dan één van de hoeken een veelvoud is van $\frac{\pi}{4}$. (Bron: ingangsexamen KMS polytechnische wetenschappen 2001)

Uit $\alpha+\beta+\gamma=\pi$ volgt dat $2\alpha+2\beta=2\pi-2\gamma$ en dus $\cos(2\alpha+2\beta)=\cos(2\pi-2\gamma)=\cos 2\gamma$.

$$\begin{aligned}
&\cos 4\alpha+\cos 4\beta+\cos 4\gamma=-1\\
&\Leftrightarrow 2\cos(2\alpha+2\beta)\cos(2\alpha-2\beta)+2\cos^2(2\alpha+2\beta)=0\\
&\Leftrightarrow 2\cos(2\alpha+2\beta)\big(\cos(2\alpha-2\beta)+\cos(2\alpha+2\beta)\big)=0\\
&\Leftrightarrow 4\cos 2\gamma\cos 2\alpha\cos 2\beta=0\\
&\Leftrightarrow \cos 2\gamma=0 \ \vee\ \cos 2\alpha=0 \ \vee\ \cos 2\beta=0\\
&\Leftrightarrow 2\gamma=\frac{\pi}{2}+k\pi \ \vee\ 2\alpha=\frac{\pi}{2}+k\pi \ \vee\ 2\beta=\frac{\pi}{2}+k\pi\\
&\Leftrightarrow \gamma=\frac{\pi}{4}+k\frac{\pi}{2} \ \vee\ \alpha=\frac{\pi}{4}+k\frac{\pi}{2} \ \vee\ \beta=\frac{\pi}{4}+k\frac{\pi}{2}
\end{aligned}$$

Maar omdat $\alpha,\beta,\gamma\in\left]0,\pi\right[$ moet $\alpha=\frac{\pi}{4}$ of $\alpha=\frac{3\pi}{4}$ of $\beta=\frac{\pi}{4}$ of $\beta=\frac{3\pi}{4}$ of $\gamma=\frac{\pi}{4}$ of $\gamma=\frac{3\pi}{4}$ □

8. Bewijs: $\forall\alpha\in\left]0,\frac{\pi}{2}\right[:\ \sqrt{1+\cos\alpha}+\sqrt{1-\cos\alpha}=2\sin\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)$

$$LL=\sqrt{1+\cos\alpha}+\sqrt{1-\cos\alpha}=\sqrt{2\cos^2\frac{\alpha}{2}}+\sqrt{2\sin^2\frac{\alpha}{2}}=\sqrt{2}\cos\frac{\alpha}{2}+\sqrt{2}\sin\frac{\alpha}{2}\quad\left(\text{omdat }\frac{\alpha}{2}\in\left]0,\frac{\pi}{4}\right[\right)$$

$$RL=2\sin\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)=2\left(\sin\frac{\pi}{4}\cos\frac{\alpha}{2}+\cos\frac{\pi}{4}\sin\frac{\alpha}{2}\right)=2\left(\frac{\sqrt{2}}{2}\cos\frac{\alpha}{2}+\frac{\sqrt{2}}{2}\sin\frac{\alpha}{2}\right)=\sqrt{2}\cos\frac{\alpha}{2}+\sqrt{2}\sin\frac{\alpha}{2}$$

9. Bewijs zonder rekenmachine dat $\sin 20°.\sin 40°.\sin 60°.\sin 80°=\frac{3}{16}$

$$\underbrace{\sin 20°.\sin 40°}_{Simpson}.\underbrace{\sin 60°}_{=\frac{\sqrt{3}}{2}}.\sin 80°=\frac{1}{2}(\cos 20°-\underbrace{\cos 60°}_{=\frac{1}{2}})\cdot\frac{\sqrt{3}}{2}\cdot\sin 80°=\frac{\sqrt{3}}{4}\underbrace{\sin 80°\cos 20°}_{Simpson}-\frac{\sqrt{3}}{8}\sin 80°$$

$$=\frac{\sqrt{3}}{4}\cdot\frac{1}{2}(\underbrace{\sin 60°}_{=\frac{\sqrt{3}}{2}}+\underbrace{\sin 100°}_{=\sin 80°})-\frac{\sqrt{3}}{8}\sin 80°=\frac{3}{16}+\cancel{\frac{\sqrt{3}}{8}\sin 80°}-\cancel{\frac{\sqrt{3}}{8}\sin 80°}=\frac{3}{16}$$

10. Bereken zonder rekenmachine: $\sin 1°.\sin 3°.\sin 5°.\sin 7°.\ \ldots\ .\sin 179°$

$$\sin 1°.\sin 3°.\sin 5°.\ \ldots\ .\sin 177°.\sin 179°=\frac{\sin 1°.\sin 2°.\sin 3°.\ \ldots\ .\sin 178°.\sin 179°}{\sin 2°.\sin 4°.\sin 6°.\ \ldots\ .\sin 178°}$$

$$=\frac{(\sin 1°.\sin 179°).(\sin 2°.\sin 178°).(\sin 3°.\sin 177°).\ \ldots\ .(\sin 89°.\sin 91°).\sin 90°}{\sin 2°.\sin 4°.\sin 6°.\ \ldots\ .\sin 178°}$$

$$=\frac{(\sin 1°.\cos 1°).(\sin 2°.\cos 2°).(\sin 3°.\cos 3°).\ \ldots\ .(\sin 89°.\cos 89°).\sin 90°}{\sin 2°.\sin 4°.\sin 6°.\ \ldots\ .\sin 178°}$$

$$=\frac{\frac{1}{2}\cancel{\sin 2°}.\frac{1}{2}\cancel{\sin 4°}.\frac{1}{2}\cancel{\sin 6°}.\ \ldots\ .\frac{1}{2}\cancel{\sin 178°}}{\cancel{\sin 2°}.\cancel{\sin 4°}.\cancel{\sin 6°}.\ \ldots\ .\cancel{\sin 178°}}=\frac{1}{2^{89}}$$

11. In driehoek ΔABC geldt dat $\sin\left(\hat{A}+\frac{\hat{B}}{2}\right)=k.\sin\frac{\hat{B}}{2}$, met $k\in\mathbb{R}_0^+$.

Bewijs dat $\frac{k-1}{k+1}=\tan\frac{\hat{A}}{2}\cdot\tan\frac{\hat{C}}{2}$.

Eerste methode: Merk op dat $k=\dfrac{\sin\left(\hat{A}+\frac{\hat{B}}{2}\right)}{\sin\frac{\hat{B}}{2}}=\dfrac{\sin\hat{A}\cos\frac{\hat{B}}{2}+\cos A\sin\frac{\hat{B}}{2}}{\sin\frac{\hat{B}}{2}}=\sin\hat{A}\cot\frac{\hat{B}}{2}+\cos\hat{A}$

Dus, dan krijgen we achtereenvolgens:

$$\frac{k-1}{k+1}=\frac{\sin\hat{A}\cot\frac{\hat{B}}{2}+\cos\hat{A}-1}{\sin\hat{A}\cot\frac{\hat{B}}{2}+\cos\hat{A}+1}=\frac{2\sin\frac{\hat{A}}{2}\cos\frac{\hat{A}}{2}\tan\frac{\hat{A}+\hat{C}}{2}-2\sin^2\frac{\hat{A}}{2}}{2\sin\frac{\hat{A}}{2}\cos\frac{\hat{A}}{2}\tan\frac{\hat{A}+\hat{C}}{2}+2\cos^2\frac{\hat{A}}{2}}$$

$$=\frac{\cancel{2}\sin\frac{\hat{A}}{2}\left(\cos\frac{\hat{A}}{2}\tan\frac{\hat{A}+\hat{C}}{2}-\sin\frac{\hat{A}}{2}\right)}{\cancel{2}\cos\frac{\hat{A}}{2}\left(\sin\frac{\hat{A}}{2}\tan\frac{\hat{A}+\hat{C}}{2}+\cos\frac{\hat{A}}{2}\right)}=\tan\frac{\hat{A}}{2}\frac{\cos\frac{\hat{A}}{2}\left(\dfrac{\tan\frac{\hat{A}}{2}+\tan\frac{\hat{C}}{2}}{1-\tan\frac{\hat{A}}{2}\tan\frac{\hat{C}}{2}}\right)-\sin\frac{\hat{A}}{2}}{\sin\frac{\hat{A}}{2}\left(\dfrac{\tan\frac{\hat{A}}{2}+\tan\frac{\hat{C}}{2}}{1-\tan\frac{\hat{A}}{2}\tan\frac{\hat{C}}{2}}\right)+\cos\frac{\hat{A}}{2}}$$

$$=\tan\frac{\hat{A}}{2}\frac{\cos\frac{\hat{A}}{2}\left(\tan\frac{\hat{A}}{2}+\tan\frac{\hat{C}}{2}\right)-\sin\frac{\hat{A}}{2}\left(1-\tan\frac{\hat{A}}{2}\tan\frac{\hat{C}}{2}\right)}{\sin\frac{\hat{A}}{2}\left(\tan\frac{\hat{A}}{2}+\tan\frac{\hat{C}}{2}\right)+\cos\frac{\hat{A}}{2}\left(1-\tan\frac{\hat{A}}{2}\tan\frac{\hat{C}}{2}\right)}$$

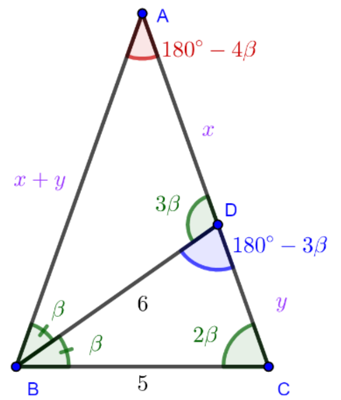
$$=\tan\frac{\hat{A}}{2}\frac{\cancel{\sin\frac{\hat{A}}{2}}+\cos\frac{\hat{A}}{2}\tan\frac{\hat{C}}{2}\cancel{-\sin\frac{\hat{A}}{2}}+\sin\frac{\hat{A}}{2}\tan\frac{\hat{A}}{2}\tan\frac{\hat{C}}{2}}{\sin\frac{\hat{A}}{2}\tan\frac{\hat{A}}{2}\cancel{+\sin\frac{\hat{A}}{2}\tan\frac{\hat{C}}{2}}+\cos\frac{\hat{A}}{2}\cancel{-\sin\frac{\hat{A}}{2}\tan\frac{\hat{C}}{2}}}$$

$$=\tan\frac{\hat{A}}{2}\frac{\cancel{\left(\cos\frac{\hat{A}}{2}+\sin\frac{\hat{A}}{2}\tan\frac{\hat{A}}{2}\right)}\tan\frac{\hat{C}}{2}}{\cancel{\sin\frac{\hat{A}}{2}\tan\frac{\hat{A}}{2}+\cos\frac{\hat{A}}{2}}}=\tan\frac{\hat{A}}{2}\tan\frac{\hat{C}}{2}\quad\square$$

Tweede methode: $\hat{A}+\hat{B}+\hat{C}=\pi \Leftrightarrow \frac{\hat{A}+\hat{B}+\hat{C}}{2}=\frac{\pi}{2} \Leftrightarrow \frac{\hat{A}+\hat{B}}{2}=\frac{\pi-\hat{C}}{2} \Rightarrow \cot\left(\frac{\hat{A}+\hat{B}}{2}\right)=\tan\frac{\hat{C}}{2}$,

dus: $$\frac{k-1}{k+1}=\frac{\frac{\sin\left(\hat{A}+\frac{\hat{B}}{2}\right)}{\sin\frac{\hat{B}}{2}}-1}{\frac{\sin\left(\hat{A}+\frac{\hat{B}}{2}\right)}{\sin\frac{\hat{B}}{2}}+1}=\frac{\sin\left(\hat{A}+\frac{\hat{B}}{2}\right)-\sin\frac{\hat{B}}{2}}{\sin\left(\hat{A}+\frac{\hat{B}}{2}\right)+\sin\frac{\hat{B}}{2}}=\frac{2\cos\left(\frac{\hat{A}+\hat{B}}{2}\right)\cdot\sin\frac{\hat{A}}{2}}{2\sin\left(\frac{\hat{A}+\hat{B}}{2}\right)\cdot\cos\frac{\hat{A}}{2}}=\underbrace{\cot\left(\frac{\hat{A}+\hat{B}}{2}\right)}_{=\tan\frac{\hat{C}}{2}}\cdot\tan\frac{\hat{A}}{2}\ \square$$

12. In een gelijkbenige driehoek ΔABC (met $|BC|=5$ en $|AB|=|AC|$) geldt dat de (binnen-)bissectrice van hoek $\hat{B}$ de zijde $[AC]$ snijdt in D, en wel zo dat $|BD|=6$. Bereken $\cos\hat{B}$.



Noem voor het gemak $\hat{B}=2\beta$. Je kan dan eenvoudig alle hoeken berekenen op de figuur in functie van β. Noem $|AD|=x$ en $|DC|=y$.

Passen we de sinusregel toe in de drie driehoeken, dan geldt:

in ΔABD: $\frac{6}{\sin 4\beta}=\frac{x}{\sin\beta}=\frac{x+y}{\sin 3\beta}$, in ΔBCD: $\frac{6}{\sin 2\beta}=\frac{y}{\sin\beta}=\frac{5}{\sin 3\beta}$ en in ΔABC: $\frac{5}{\sin 4\beta}=\frac{x+y}{\sin 2\beta}$.

Uit de eerste haal je $x=\frac{6\sin\beta}{\sin 4\beta}$, uit de tweede $y=\frac{6\sin\beta}{\sin 2\beta}$ en uit de laatste $x+y=\frac{5\sin 2\beta}{\sin 4\beta}$.

Combineren geeft ons de vergelijking $\frac{6\sin\beta}{\sin 4\beta}+\frac{6\sin\beta}{\sin 2\beta}=\frac{5\sin 2\beta}{\sin 4\beta} \Leftrightarrow \frac{6\sin\beta}{\sin 4\beta}+\frac{6\sin\beta}{\sin 2\beta}=\frac{10\sin\beta\cos\beta}{\sin 4\beta}$

$$\Leftrightarrow 6+\frac{\overbrace{6\sin 4\beta}^{=12\sin 2\beta\cos 2\beta}}{\sin 2\beta}=10\cos\beta \Leftrightarrow 6\overbrace{\cos 2\beta}^{=2\cos^2\beta-1}-5\cos\beta+3=0 \Leftrightarrow 12\cos^2\beta-5\cos\beta-3=0$$

Hieruit volgt ($\Delta=169$): $\cos\beta=\frac{5\pm 13}{24}$ $\nearrow 3/4$, $\searrow -1/3$, maar omdat β sowieso scherp is moet $\cos\beta=3/4$.

Dus vinden we dat: $\cos\hat{B}=\cos 2\beta=2\cos^2\beta-1=2\cdot\frac{9}{16}-1=\frac{1}{8}$.

13. Bereken zonder rekenmachine: $\frac{\cos^3 15°+\sin^3 15°}{\cos 15°+\sin 15°}$.

$$\frac{\cos^3 15°+\sin^3 15°}{\cos 15°+\sin 15°}=\frac{(\cos 15°+\sin 15°)\cdot(\cos^2 15°-\cos 15°\sin 15°+\sin^2 15°)}{\cos 15°+\sin 15°}=1-\frac{1}{2}\sin 30°=1-\frac{1}{4}=\frac{3}{4}$$