

1. Los de vergelijkingen op:

a) ★	$2\cos\left(2x + \frac{\pi}{4}\right) + \sqrt{3} = 0 \Leftrightarrow \cos\left(2x + \frac{\pi}{4}\right) = -\frac{\sqrt{3}}{2} \Leftrightarrow \cos\left(2x + \frac{\pi}{4}\right) = \cos\frac{5\pi}{6}$ $\Leftrightarrow 2x + \frac{\pi}{4} = \frac{5\pi}{6} + 2k\pi \vee 2x + \frac{\pi}{4} = -\frac{5\pi}{6} + 2k\pi \Leftrightarrow x = \frac{7\pi}{24} + k\pi \vee x = -\frac{13\pi}{24} + k\pi$
b) ★	$\tan 2x = -\cot x \Leftrightarrow \tan 2x = \cot(-x) \Leftrightarrow \tan 2x = \tan\left(\frac{\pi}{2} + x\right) \Leftrightarrow 2x = \frac{\pi}{2} + x + k\pi \Leftrightarrow x = \frac{\pi}{2} + k\pi$
c) ★	$\sin x \cdot \cos x = \frac{1}{4} \Leftrightarrow \frac{\sin 2x}{2} = \frac{1}{4} \Leftrightarrow \sin 2x = \frac{1}{2} \Leftrightarrow \sin 2x = \sin\frac{\pi}{6}$ $\Leftrightarrow 2x = \frac{\pi}{6} + 2k\pi \vee 2x = \frac{5\pi}{6} + 2k\pi \Leftrightarrow x = \frac{\pi}{12} + k\pi \vee x = \frac{5\pi}{12} + k\pi$
d) ★★	$\cos x \cdot \cos 3x = \sin 2x \cdot \sin 4x \Leftrightarrow \frac{1}{2}(\cos 2x + \cos 4x) = \frac{1}{2}(\cos 2x - \cos 6x) \Leftrightarrow \cos 4x = -\cos 6x$ $\Leftrightarrow \cos 4x = \cos(\pi - 6x) \Leftrightarrow 4x = \pi - 6x + 2k\pi \vee 4x = 6x - \pi + 2k\pi$ $\Leftrightarrow x = \frac{\pi}{10} + \frac{k\pi}{5} \vee x = \frac{\pi}{2} + k\pi$
e) ★★	$\cos x + 5\sin\frac{x}{2} = 3 \Leftrightarrow 1 - 2\sin^2\frac{x}{2} + 5\sin\frac{x}{2} = 3 \Leftrightarrow 2\sin^2\frac{x}{2} - 5\sin\frac{x}{2} + 2 = 0$ $\stackrel{\Delta=9}{\Leftrightarrow} \sin\frac{x}{2} = \frac{1}{2} \vee \cancel{\sin\frac{x}{2} = 2} \Leftrightarrow \sin\frac{x}{2} = \sin\frac{\pi}{6} \Leftrightarrow \frac{x}{2} = \frac{\pi}{6} + 2k\pi \vee \frac{x}{2} = \frac{5\pi}{6} + 2k\pi$ $\Leftrightarrow x = \frac{\pi}{3} + 4k\pi \vee x = \frac{5\pi}{3} + 4k\pi$
f) ★★	$\sin 4x + 4\sin x = 8\sin^3 x \Leftrightarrow 2\sin 2x \cos 2x + 4\sin x - 8\sin^3 x = 0$ $\Leftrightarrow 4\sin x \cos x \cos 2x + 4\sin x - 8\sin^3 x = 0 \Leftrightarrow 4\sin x (\cos x \cos 2x + 1 - 2\sin^2 x) = 0$ $\Leftrightarrow \sin x = 0 \vee \cos x (2\cos^2 x - 1) + 1 - 2(1 - \cos^2 x) = 0 \stackrel{c=\cos x}{\Leftrightarrow} x = k\pi \vee 2c^3 + 2c^2 - c - 1 = 0$ $\stackrel{Ho}{\Leftrightarrow} x = k\pi \vee c = -1 \vee c = -\frac{\sqrt{2}}{2} \vee c = \frac{\sqrt{2}}{2} \Leftrightarrow x = k\pi \vee \cos x = \cos \pi \vee \cos x = \cos \frac{3\pi}{4} \vee \cos x = \cos \frac{\pi}{4}$ $\Leftrightarrow x = k\pi \vee x = \pi + 2k\pi \vee x = \frac{3\pi}{4} + 2k\pi \vee x = -\frac{3\pi}{4} + 2k\pi \vee x = \frac{\pi}{4} + 2k\pi \vee x = -\frac{\pi}{4} + 2k\pi$
g) ★	$5\cos^3 x - 2\cos x - 3 = 0 \stackrel{Ho}{\Leftrightarrow} \cos x = 1 \vee \underbrace{5\cos^2 x + 5\cos x + 3 = 0}_{\Delta < 0} \Leftrightarrow \cos x = \cos 0 \Leftrightarrow x = 2k\pi$
h) ★★	$\tan^2 x + 1 = (3 - \sin 2x) \cdot \tan x \stackrel{t=\tan x}{\Leftrightarrow} t^2 + 1 = \left(3 - \frac{2t}{1+t^2}\right) \cdot t \Leftrightarrow t^4 - 3t^3 + 4t^2 - 3t + 1 = 0$ $\stackrel{Ho}{\Leftrightarrow} t = 1 \vee \cancel{t^2 - t + 1 = 0} \Leftrightarrow \tan x = \tan \frac{\pi}{4} \Leftrightarrow x = \frac{\pi}{4} + k\pi$
i) ★★	$\sin^4 x = 3\sin^2 x \cdot \cos^2 x + 4\cos^4 x \Leftrightarrow \frac{\sin^4 x}{\cos^4 x} = \frac{3\sin^2 x \cdot \cos^2 x + 4\cos^4 x}{\cos^4 x} \Leftrightarrow \tan^4 x = 3\tan^2 x + 4$ $\Leftrightarrow \tan^2 x = -1 \vee \tan^2 x = 4 \Leftrightarrow \tan x = 2 \vee \tan x = -2 \Leftrightarrow x = Bgtan 2 + k\pi \vee x = Bgtan(-2) + k\pi$
j) ★★	$\sin^3 x + 2\sin x \cdot \cos^2 x + 3\cos x = 3\cos x \cdot \sin^2 x$ $\Leftrightarrow \sin^3 x + 2\sin x \cos^2 x + 3\cos x (\sin^2 x + \cos^2 x) = 3\cos x \cdot \sin^2 x$ $\Leftrightarrow \sin^3 x + 2\sin x \cos^2 x + 3\cos^3 x = 0 \Leftrightarrow \tan^3 x + 2\tan x + 3 = 0$ $\Leftrightarrow \tan x = -1 \vee \underbrace{\tan^2 x - \tan x + 3 = 0}_{\Delta < 0} \Leftrightarrow \tan x = \tan\left(-\frac{\pi}{4}\right) \Leftrightarrow x = -\frac{\pi}{4} + k\pi$

$$\cos^4 x + \sin^4 x = \frac{1}{2} \Leftrightarrow (\cos^2 x + \sin^2 x)^2 - 2\cos^2 x \sin^2 x = \frac{1}{2} \Leftrightarrow \cos^2 x \sin^2 x = \frac{1}{4} \Leftrightarrow \frac{\sin^2 2x}{4} = \frac{1}{4}$$

k) ★★

$$\Leftrightarrow \sin^2 2x = 1 \Leftrightarrow \sin 2x = 1 \vee \sin 2x = -1 \Leftrightarrow 2x = \frac{\pi}{2} + 2k\pi \vee 2x = -\frac{\pi}{2} + 2k\pi$$

$$\Leftrightarrow x = \frac{\pi}{4} + k\pi \vee x = -\frac{\pi}{4} + k\pi \Leftrightarrow x = \frac{\pi}{4} + k\frac{\pi}{2}$$

$$\sin x + \cos x = \frac{6}{\sec x + \csc x} \Leftrightarrow \sin x + \cos x = \frac{6}{\frac{1}{\cos x} + \frac{1}{\sin x}} \Leftrightarrow \sin x + \cos x = \frac{6 \cos x \sin x}{\sin x + \cos x}$$

l) ★★★

$$\Leftrightarrow (\sin x + \cos x)^2 = 6 \cos x \sin x \Leftrightarrow 1 + \sin 2x = 3 \sin 2x \Leftrightarrow \sin 2x = \frac{1}{2} \Leftrightarrow \sin 2x = \sin \frac{\pi}{6}$$

$$\Leftrightarrow 2x = \frac{\pi}{6} + 2k\pi \vee 2x = \frac{5\pi}{6} + 2k\pi \Leftrightarrow x = \frac{\pi}{12} + k\pi \vee x = \frac{5\pi}{12} + k\pi$$

$$\sqrt{3} \cos x + \sqrt{3} = \sin x \Leftrightarrow \sin x - \sqrt{3} \cos x = \sqrt{3} \Leftrightarrow \sin x - \tan \frac{\pi}{3} \cos x = \sqrt{3}$$

m) ★★

$$\Leftrightarrow \sin x \cos \frac{\pi}{3} - \sin \frac{\pi}{3} \cos x = \sqrt{3} \cos \frac{\pi}{3} \Leftrightarrow \sin \left( x - \frac{\pi}{3} \right) = \frac{\sqrt{3}}{2} \Leftrightarrow \sin \left( x - \frac{\pi}{3} \right) = \sin \frac{\pi}{3}$$

$$\Leftrightarrow x - \frac{\pi}{3} = \frac{\pi}{3} + 2k\pi \vee x - \frac{\pi}{3} = \frac{2\pi}{3} + 2k\pi \Leftrightarrow x = \frac{2\pi}{3} + 2k\pi \vee x = \pi + 2k\pi$$

$$3 \tan x = \frac{5}{\cos x} + 4 \Leftrightarrow 3 \sin x = 5 + 4 \cos x \Leftrightarrow \sin x - \frac{4}{3} \cos x = \frac{5}{3} \Leftrightarrow \sin x - \tan \varphi \cos x = \frac{5}{3}$$

n) ★★★

$$\Leftrightarrow \sin x \cos \varphi - \sin \varphi \cos x = \frac{5}{3} \cdot \cos \varphi \stackrel{**}{\Leftrightarrow} \sin(x - \varphi) = 1 \Leftrightarrow \sin(x - \varphi) = \sin \frac{\pi}{2}$$

$$\Leftrightarrow x - \varphi = \frac{\pi}{2} + 2k\pi \Leftrightarrow x = \frac{\pi}{2} + \text{Bgtan} \frac{4}{3} + 2k\pi \quad (**: \tan \varphi = \frac{4}{3} \Rightarrow \text{neem cos} \varphi = \frac{3}{5})$$

$$2 \sin^2 x + \sin 2x = \cos 2x + 1 \Leftrightarrow 1 - \cos 2x + \sin 2x = \cos 2x + 1 \Leftrightarrow \sin 2x = 2 \cos 2x \Leftrightarrow \tan 2x = 2$$

o) ★★

$$\Leftrightarrow 2x = \text{Bgtan} 2 + k\pi \Leftrightarrow x = \frac{\text{Bgtan} 2}{2} + k\frac{\pi}{2}$$

$$\sin^4 x + \cos^4 x = \sin x \cdot \cos x \Leftrightarrow (\sin^2 x + \cos^2 x)^2 - 2 \sin^2 x \cos^2 x = \sin x \cos x$$

p) ★★

$$\Leftrightarrow 1 - \frac{\sin^2 2x}{2} = \frac{\sin 2x}{2} \Leftrightarrow \sin^2 2x + \sin 2x - 2 = 0 \Leftrightarrow \sin 2x = 1 \vee \cancel{\sin 2x = -2}$$

$$\Leftrightarrow \sin 2x = \sin \frac{\pi}{2} \Leftrightarrow 2x = \frac{\pi}{2} + 2k\pi \Leftrightarrow x = \frac{\pi}{4} + k\pi$$

$$11 \cos 2x - 7 \sin 2x + 13 = 0 \Leftrightarrow \sin 2x - \frac{11}{7} \cos 2x = \frac{13}{7} \Leftrightarrow \sin 2x - \tan \varphi \cos 2x = \frac{13}{7}$$

$$\Leftrightarrow \sin 2x \cos \varphi - \sin \varphi \cos 2x = \frac{13}{7} \cos \varphi \Leftrightarrow \sin(2x - \varphi) = \frac{13}{\sqrt{170}}$$

q) ★★

$$\Leftrightarrow 2x - \varphi = \text{Bgsin} \frac{13}{\sqrt{170}} + 2k\pi \vee 2x - \varphi = \pi - \text{Bgsin} \frac{13}{\sqrt{170}} + 2k\pi$$

$$\Leftrightarrow x = \frac{\text{Bgsin} \frac{13}{\sqrt{170}} + \text{Bgtan} \frac{11}{7}}{2} + k\pi \vee x = \frac{\pi - \text{Bgsin} \frac{13}{\sqrt{170}} + \text{Bgtan} \frac{11}{7}}{2} + k\pi$$

Alternatief: stel  $t = \tan x$ , dan is  $\sin 2x = \frac{2t}{1+t^2}$  en  $\cos 2x = \frac{1-t^2}{1+t^2}$ , zodat

$$11\cos 2x - 7\sin 2x + 13 = 0 \Leftrightarrow 11\frac{1-t^2}{1+t^2} - 7\frac{2t}{1+t^2} + 13 = 0 \Leftrightarrow 11(1-t^2) - 14t + 13(1+t^2) = 0$$

$$\Leftrightarrow 2t^2 - 14t + 24 = 0 \Leftrightarrow t = 3 \vee t = 4 \Leftrightarrow \tan x = 3 \vee \tan x = 4$$

$$\Leftrightarrow x = Bgtan3 + k\pi \vee x = Bgtan4 + k\pi$$

(Reken zelf eens na dat dit wel degelijk dezelfde antwoorden zijn)

$$\frac{\tan x}{\tan 2x} = \frac{\tan 2x}{\tan x} \stackrel{x \neq \frac{k\pi}{2}}{\Leftrightarrow} \tan^2 x = \tan^2 2x \Leftrightarrow \tan x = \tan 2x \vee \tan x = \overbrace{-\tan 2x}^{=\tan(-2x)}$$

$$\Leftrightarrow x = 2x + k\pi \vee x = -2x + k\pi \Leftrightarrow \cancel{x=k\pi} \vee x = k \frac{\pi}{3}$$

Dus  $x$  moet een veelvoud zijn van  $\frac{\pi}{3}$  maar mag geen veelvoud zijn van  $\frac{\pi}{2}$ . De eenvoudigste manier om

$$\text{dit te schrijven is: } V = \left\{ \frac{\pi}{3} + k\pi; \frac{2\pi}{3} + k\pi \right\}$$

$$2\cos 6x = 2(\sqrt{3} + \sqrt{2})\sin 3x + \sqrt{6} + 2 \Leftrightarrow 2(1 - 2\sin^2 3x) = 2(\sqrt{3} + \sqrt{2})\sin 3x + \sqrt{6} + 2$$

$$\Leftrightarrow 4\sin^2 3x + 2(\sqrt{3} + \sqrt{2})\sin 3x + \sqrt{6} = 0 \Leftrightarrow \sin 3x = -\frac{\sqrt{2}}{2} \vee \sin 3x = \frac{\sqrt{3}}{2}$$

$$\Delta = 4(5+2\sqrt{6}) - 16\sqrt{6} = [2(\sqrt{3}-\sqrt{2})]^2$$

$$s) \quad \star \star \star \quad \Leftrightarrow \sin 3x = \sin\left(-\frac{\pi}{4}\right) \vee \sin 3x = \sin\left(-\frac{\pi}{3}\right)$$

$$\Leftrightarrow 3x = -\frac{\pi}{4} + 2k\pi \vee 3x = -\frac{3\pi}{4} + 2k\pi \vee 3x = -\frac{\pi}{3} + 2k\pi \vee 3x = -\frac{2\pi}{3} + 2k\pi$$

$$\Leftrightarrow x = -\frac{\pi}{12} + \frac{2}{3}k\pi \vee x = -\frac{\pi}{4} + \frac{2}{3}k\pi \vee x = -\frac{\pi}{9} + \frac{2}{3}k\pi \vee x = -\frac{2\pi}{9} + \frac{2}{3}k\pi$$

$$t) \quad \star \quad 2 + \cos 4x + \cos 2x = 0 \Leftrightarrow 2 + 2\cos^2 2x - 1 + \cos 2x = 0 \Leftrightarrow \overbrace{2\cos^2 2x + \cos 2x + 1 = 0}^{\Delta < 0}$$

$$\tan 6x - \tan 5x = \frac{1}{2}\sin x \Leftrightarrow \frac{\sin 6x}{\cos 6x} - \frac{\sin 5x}{\cos 5x} = \frac{1}{2}\sin x$$

$$\Leftrightarrow \sin 6x \cos 5x - \sin 5x \cos 6x = \frac{1}{2}\sin x \cos 6x \cos 5x \Leftrightarrow \sin(6x - 5x) = \frac{1}{2}\sin x \cos 6x \cos 5x$$

$$u) \quad \star \star \star \quad \Leftrightarrow \sin x = \frac{1}{2}\sin x \cos 6x \cos 5x \Leftrightarrow \sin x \left(1 - \frac{1}{2}\cos 6x \cos 5x\right) = 0$$

$$\Leftrightarrow \sin x = 0 \vee 1 - \frac{1}{2}\cos 6x \cos 5x = 0 \Leftrightarrow x = k\pi \vee \cancel{\cos 6x \cos 5x = 2}$$

$$4(\cos^3 x - \sin^3 x) = 5(\cos x - \sin x)$$

$$\Leftrightarrow 4(\cos x - \sin x)(\cos^2 x + \cos x \sin x + \sin^2 x) = 5(\cos x - \sin x)$$

$$v) \quad \star \star \quad \Leftrightarrow \cos x - \sin x = 0 \vee 4\left(1 + \frac{1}{2}\sin 2x\right) = 5 \Leftrightarrow \tan x = 1 \vee \sin 2x = \frac{1}{2}$$

$$\Leftrightarrow \tan x = \tan \frac{\pi}{4} \vee \sin 2x = \sin \frac{\pi}{6} \Leftrightarrow x = \frac{\pi}{4} + k\pi \vee 2x = \frac{\pi}{6} + 2k\pi \vee 2x = \frac{5\pi}{6} + 2k\pi$$

$$\Leftrightarrow x = \frac{\pi}{4} + k\pi \vee x = \frac{\pi}{12} + k\pi \vee x = \frac{5\pi}{12} + k\pi$$

$$\begin{aligned}
 \frac{\sin x + 1}{\cos x} = \frac{\cos x + 1}{\sin x} &\Leftrightarrow \sin^2 x + \sin x = \cos^2 x + \cos x \Leftrightarrow \sin^2 x - \cos^2 x = \cos x - \sin x \\
 \Leftrightarrow (\sin x - \cos x)(\sin x + \cos x) &= -(\sin x - \cos x) \Leftrightarrow \sin x - \cos x = 0 \vee \sin x + \cos x = -1 \\
 \Leftrightarrow \tan x = 1 \vee \frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2} &= -1 \Leftrightarrow \tan x = \tan \frac{\pi}{4} \vee 1+2t-t^2 = -1-t^2 \Leftrightarrow x = \frac{\pi}{4} + k\pi \vee t = -1 \\
 \Leftrightarrow x = \frac{\pi}{4} + k\pi \vee \tan \frac{x}{2} = \tan \frac{-\pi}{4} &\Leftrightarrow x = \frac{\pi}{4} + k\pi \vee x = -\frac{\pi}{2} + 2k\pi
 \end{aligned}$$

w) ★★\*

Ook OPGELET als je voor de tweede vergelijking de andere methode toepast:

$$\begin{aligned}
 \sin x + \cos x = -1 &\Leftrightarrow \sin x + \tan \frac{\pi}{4} \cos x = -1 \Leftrightarrow \sin x \cos \frac{\pi}{4} + \sin \frac{\pi}{4} \cos x = -\cos \frac{\pi}{4} \\
 \Leftrightarrow \sin \left( x + \frac{\pi}{4} \right) &= \sin \left( -\frac{\pi}{4} \right) \Leftrightarrow x + \frac{\pi}{4} = -\frac{\pi}{4} + 2k\pi \vee x + \frac{\pi}{4} = \frac{-3\pi}{4} + 2k\pi \\
 \Leftrightarrow x = -\frac{\pi}{2} + 2k\pi \vee x = -\frac{\pi}{4} + 2k\pi &\quad \boxed{BV}
 \end{aligned}$$

$$\begin{aligned}
 \underline{\sin x + \sin 2x} + \underline{\sin 3x} &= \underline{\cos x + \cos 2x} + \underline{\cos 3x} \Leftrightarrow 2\sin 2x \cos x + \sin 2x = 2\cos 2x \cos x + \cos 2x \\
 \Leftrightarrow \sin 2x(2\cos x + 1) &= \cos 2x(2\cos x + 1) \Leftrightarrow 2\cos x + 1 = 0 \vee \sin 2x = \cos 2x
 \end{aligned}$$

x) ★★

$$\begin{aligned}
 \Leftrightarrow \cos x = -\frac{1}{2} \vee \tan 2x = 1 &\Leftrightarrow \cos x = \cos \frac{2\pi}{3} \vee \tan 2x = \tan \frac{\pi}{4} \\
 \Leftrightarrow x = \frac{2\pi}{3} + 2k\pi \vee x = -\frac{2\pi}{3} + 2k\pi &\vee x = \frac{\pi}{8} + k\frac{\pi}{2}
 \end{aligned}$$

$$\sqrt{1+\cos x} = \sqrt{2} \sin \frac{x}{2} \stackrel{\text{III}}{\Rightarrow} 1+\cos x = 2\sin^2 \frac{x}{2} \Leftrightarrow 1+\cos x = 1-\cos x \Leftrightarrow \cos x = 0 \Leftrightarrow x = \frac{\pi}{2} + k\pi$$

y) ★★★

Maar wegens de kwadrateringsvoorwaarde moet  $\sin \frac{x}{2} > 0$ , dus de oplossingen worden gegeven door:

$$V = \left\{ \frac{\pi}{2} + [4]k\pi, \frac{3\pi}{2} + [4]k\pi \right\}$$

2. Los op:

$$\begin{aligned} \sin(4\sin x) = \cos(5\cos x) &\Leftrightarrow \cos(5\cos x) = \cos\left(\frac{\pi}{2} - 4\sin x\right) \\ \Leftrightarrow 5\cos x &= \frac{\pi}{2} - 4\sin x + 2k\pi \quad \vee \quad 5\cos x = -\frac{\pi}{2} + 4\sin x + 2k\pi \\ \Leftrightarrow 4\sin x + 5\cos x &= \frac{\pi}{2} + 2k\pi \quad \vee \quad 4\sin x - 5\cos x = \frac{\pi}{2} + 2k\pi \end{aligned}$$

We lossen de linkse vergelijking op:

$$\text{Deze vergelijking is oplosbaar als en slechts als } 4^2 + 5^2 \leq \left(\frac{\pi}{2} + 2k\pi\right)^2 \Leftrightarrow k = 0 \vee k = -1$$

Geval 1:  $k = 0$

$$\begin{aligned} 4\sin x + 5\cos x &= \frac{\pi}{2} \stackrel{t=\tan \frac{x}{2}}{\Leftrightarrow} 4 \cdot \frac{2t}{1+t^2} + 5 \cdot \frac{1-t^2}{1+t^2} = \frac{\pi}{2} \Leftrightarrow (\pi+10)t^2 - 16t + \pi - 10 = 0 \\ \Leftrightarrow t &= \frac{8 + \sqrt{164 - \pi^2}}{\pi + 10} \quad \vee \quad t = \frac{8 - \sqrt{164 - \pi^2}}{\pi + 10} \\ \Leftrightarrow x_1 &= 2Bgtan\left(\frac{8 + \sqrt{164 - \pi^2}}{\pi + 10}\right) + 2k\pi \quad \vee \quad x_2 = 2Bgtan\left(\frac{8 - \sqrt{164 - \pi^2}}{\pi + 10}\right) + 2k\pi \end{aligned}$$

Geval 2:  $k = -1$

$$\begin{aligned} 4\sin x + 5\cos x &= \frac{-3\pi}{2} \stackrel{t=\tan \frac{x}{2}}{\Leftrightarrow} 4 \cdot \frac{2t}{1+t^2} + 5 \cdot \frac{1-t^2}{1+t^2} = \frac{-3\pi}{2} \Leftrightarrow (-3\pi+10)t^2 - 16t - 3\pi - 10 = 0 \\ \Leftrightarrow t &= \frac{8 + \sqrt{164 - 9\pi^2}}{-3\pi + 10} \quad \vee \quad t = \frac{8 - \sqrt{164 - 9\pi^2}}{-3\pi + 10} \\ \Leftrightarrow x_3 &= 2Bgtan\left(\frac{8 + \sqrt{164 - 9\pi^2}}{-3\pi + 10}\right) + 2k\pi \quad \vee \quad x_4 = 2Bgtan\left(\frac{8 - \sqrt{164 - 9\pi^2}}{-3\pi + 10}\right) + 2k\pi \end{aligned}$$

De rechtse vergelijking oplossen is niet nodig... want stel je  $x = \pi - x'$  in de eerste vergelijking dan krijg je de tweede vergelijking. De oplossingen van de tweede vergelijking zijn dus de supplementen van de eerste vergelijking. Samengevat krijgen we dus:

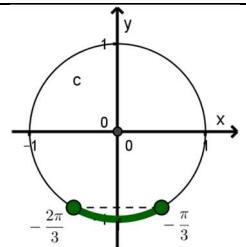
$$V = \{x_1, x_2, x_3, x_4, \pi - x_1, \pi - x_2, \pi - x_3, \pi - x_4\}$$

3. Los de volgende ongelijkheden op:

$$\sin 2x < -\frac{\sqrt{3}}{2} \Leftrightarrow -\frac{2\pi}{3} + 2k\pi < 2x < -\frac{\pi}{3} + 2k\pi$$

a)  $\star\star$

$$\Leftrightarrow -\frac{\pi}{3} + k\pi < x < -\frac{\pi}{6} + k\pi$$

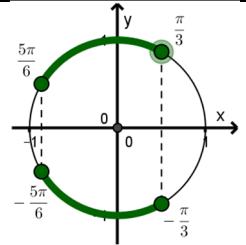


$$-\frac{\sqrt{3}}{2} \leq \cos \frac{x}{2} \leq \frac{1}{2}$$

b)  $\star\star$

$$\Leftrightarrow \frac{\pi}{3} + 2k\pi \leq \frac{x}{2} \leq \frac{5\pi}{6} + 2k\pi \vee -\frac{5\pi}{6} + 2k\pi \leq \frac{x}{2} \leq -\frac{\pi}{3} + 2k\pi$$

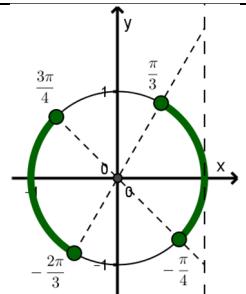
$$\Leftrightarrow \frac{2\pi}{3} + 4k\pi \leq x \leq \frac{5\pi}{3} + 4k\pi \vee -\frac{5\pi}{3} + 4k\pi \leq x \leq -\frac{2\pi}{3} + 4k\pi$$



$$-1 \leq \tan\left(\frac{2}{3}\pi - x\right) \leq \sqrt{3} \Leftrightarrow -\frac{\pi}{4} + k\pi \leq \frac{2}{3}\pi - x \leq \frac{\pi}{3} + k\pi$$

c)  $\star\star\star$

$$\Leftrightarrow -\frac{\pi}{3} + k\pi \leq x - \frac{2}{3}\pi \leq \frac{\pi}{4} + k\pi \Leftrightarrow \frac{\pi}{3} + k\pi \leq x \leq \frac{11\pi}{12} + k\pi$$

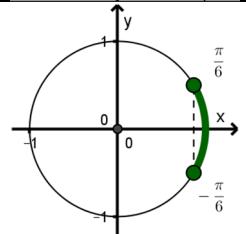


$$2\sin^2 2x + 3\sqrt{3} \cos 2x - 5 > 0 \Leftrightarrow 2(1 - \cos^2 2x) + 3\sqrt{3} \cos 2x - 5 > 0$$

d)  $\star\star$

$$\Leftrightarrow -2\cos^2 2x + 3\sqrt{3} \cos 2x - 3 > 0 \Leftrightarrow \frac{\sqrt{3}}{2} < \cos 2x < \sqrt{3}$$

$$\Leftrightarrow -\frac{\pi}{6} + 2k\pi < 2x < \frac{\pi}{6} + 2k\pi \Leftrightarrow -\frac{\pi}{12} + k\pi < x < \frac{\pi}{12} + k\pi$$

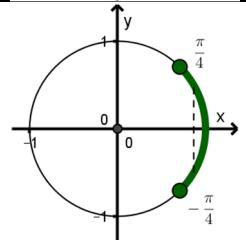


e)

$$\tan x \cdot \tan \frac{x}{2} \geq 0 \quad \boxed{t = \tan \frac{x}{2}} \quad \Leftrightarrow \frac{2t}{1-t^2} \cdot t \geq 0 \Leftrightarrow \frac{2t^2}{1-t^2} \geq 0$$

$$\Leftrightarrow -1 < t < 1 \Leftrightarrow -1 < \tan \frac{x}{2} < 1 \Leftrightarrow -\frac{\pi}{4} + k\pi < \frac{x}{2} < \frac{\pi}{4} + k\pi$$

$$\Leftrightarrow -\frac{\pi}{2} + 2k\pi < x < \frac{\pi}{2} + 2k\pi$$



$$\sin 2x < \cos x \Leftrightarrow 2\sin x \cos x - \cos x < 0 \Leftrightarrow \cos x(2\sin x - 1) < 0$$

De functie in het linkerlid heeft periode  $P = 2\pi$ .

We bekijken het tekenverloop binnen één periode  $[0, 2\pi]$ :

f)  $\star\star\star$

$x$	0	$\pi/6$	$\pi/2$	$5\pi/6$	$3\pi/2$	$2\pi$
$2\sin x - 1$	-	0	+	+	0	-
$\cos x$	+	+	0	-	-	0
$LL$	-	0	+	0	+	-

Dus:  $-\frac{\pi}{2} + 2k\pi < x < \frac{\pi}{6} + 2k\pi \vee \frac{\pi}{2} + 2k\pi < x < \frac{5\pi}{6} + 2k\pi$