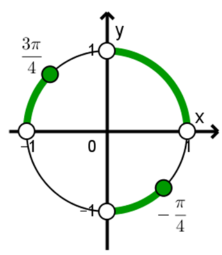
1. Los de vergelijkingen op:

a)	★	$2\cos\left(2x+\frac{\pi}{4}\right)+\sqrt{3}=0 \Leftrightarrow \cos\left(2x+\frac{\pi}{4}\right)=-\frac{\sqrt{3}}{2} \Leftrightarrow \cos\left(2x+\frac{\pi}{4}\right)=\cos\frac{5\pi}{6}$ $\Leftrightarrow 2x+\frac{\pi}{4}=\frac{5\pi}{6}+2k\pi \ \vee\ 2x+\frac{\pi}{4}=-\frac{5\pi}{6}+2k\pi \Leftrightarrow x=\frac{7\pi}{24}+k\pi \ \vee\ x=-\frac{13\pi}{24}+k\pi$
b)	★	$\tan 2x=-\cot x \Leftrightarrow \tan 2x=\cot(-x) \Leftrightarrow \tan 2x=\tan\left(\frac{\pi}{2}+x\right) \Leftrightarrow 2x=\frac{\pi}{2}+x+k\pi \Leftrightarrow x=\frac{\pi}{2}+k\pi$
c)	★	$\sin x.\cos x=\frac{1}{4} \Leftrightarrow \frac{\sin 2x}{2}=\frac{1}{4} \Leftrightarrow \sin 2x=\frac{1}{2} \Leftrightarrow \sin 2x=\sin\frac{\pi}{6}$ $\Leftrightarrow 2x=\frac{\pi}{6}+2k\pi \ \vee\ 2x=\frac{5\pi}{6}+2k\pi \Leftrightarrow x=\frac{\pi}{12}+k\pi \ \vee\ x=\frac{5\pi}{12}+k\pi$
d)	★⋆	$\cos x.\cos 3x=\sin 2x.\sin 4x \Leftrightarrow \frac{1}{2}(\cos 2x+\cos 4x)=\frac{1}{2}(\cos 2x-\cos 6x) \Leftrightarrow \cos 4x=-\cos 6x$ $\Leftrightarrow \cos 4x=\cos(\pi-6x) \Leftrightarrow 4x=\pi-6x+2k\pi \ \vee\ 4x=6x-\pi+2k\pi$ $\Leftrightarrow x=\frac{\pi}{10}+\frac{k\pi}{5} \ \vee\ x=\frac{\pi}{2}+k\pi$
e)	★⋆	$\cos x+5\sin\frac{x}{2}=3 \Leftrightarrow 1-2\sin^2\frac{x}{2}+5\sin\frac{x}{2}=3 \Leftrightarrow 2\sin^2\frac{x}{2}-5\sin\frac{x}{2}+2=0$ $\overset{\Delta=9}{\Leftrightarrow} \sin\frac{x}{2}=\frac{1}{2} \vee \cancel{\sin\frac{x}{2}=2} \Leftrightarrow \sin\frac{x}{2}=\sin\frac{\pi}{6} \Leftrightarrow \frac{x}{2}=\frac{\pi}{6}+2k\pi \ \vee\ \frac{x}{2}=\frac{5\pi}{6}+2k\pi$ $\Leftrightarrow x=\frac{\pi}{3}+4k\pi \ \vee\ x=\frac{5\pi}{3}+4k\pi$
f)	★★	$\sin 4x+4\sin x=8\sin^3 x \Leftrightarrow 2\sin 2x\cos 2x+4\sin x-8\sin^3 x=0$ $\Leftrightarrow 4\sin x\cos x\cos 2x+4\sin x-8\sin^3 x=0 \Leftrightarrow 4\sin x\left(\cos x\cos 2x+1-2\sin^2 x\right)=0$ $\Leftrightarrow \sin x=0 \ \vee\ \cos x\left(2\cos^2 x-1\right)+1-2\left(1-\cos^2 x\right)=0 \overset{c=\cos x}{\Leftrightarrow} x=k\pi \ \vee\ 2c^3+2c^2-c-1=0$ $\overset{Ho}{\Leftrightarrow} x=k\pi \vee c=-1 \vee c=-\frac{\sqrt{2}}{2} \vee c=\frac{\sqrt{2}}{2} \Leftrightarrow x=k\pi \vee \cos x=\cos\pi \vee \cos x=\cos\frac{3\pi}{4} \vee \cos x=\cos\frac{\pi}{4}$ $\Leftrightarrow x=k\pi \ \vee\ x=\pi+2k\pi \ \vee\ x=\frac{3\pi}{4}+2k\pi \ \vee\ x=\frac{-3\pi}{4}+2k\pi \ \vee\ x=\frac{\pi}{4}+2k\pi \ \vee\ x=-\frac{\pi}{4}+2k\pi$
g)	★	$5\cos^3 x-2\cos x-3=0 \overset{Ho}{\Leftrightarrow} \cos x=1 \ \vee\ \underbrace{5\cos^2 x+5\cos x+3=0}_{\Delta<0} \Leftrightarrow \cos x=\cos 0 \Leftrightarrow x=2k\pi$
h)	★⋆	$\tan^2 x+1=(3-\sin 2x).\tan x \overset{t=\tan x}{\Leftrightarrow} t^2+1=\left(3-\frac{2t}{1+t^2}\right)\cdot t \Leftrightarrow t^4-3t^3+4t^2-3t+1=0$ $\overset{Ho}{\Leftrightarrow} t=1 \vee \cancel{t^2-t+1=0} \Leftrightarrow \tan x=\tan\frac{\pi}{4} \Leftrightarrow x=\frac{\pi}{4}+k\pi$
i)	★⋆	$\sin^4 x=3\sin^2 x.\cos^2 x+4\cos^4 x \Leftrightarrow \frac{\sin^4 x}{\cos^4 x}=\frac{3\sin^2 x.\cos^2 x+4\cos^4 x}{\cos^4 x} \Leftrightarrow \tan^4 x=3\tan^2 x+4$ $\Leftrightarrow \tan^2 x=-1 \vee \tan^2 x=4 \Leftrightarrow \tan x=2 \vee \tan x=-2 \Leftrightarrow x=\text{Bgtan}2+k\pi \vee x=\text{Bgtan}(-2)+k\pi$
j)	★★	$\sin^3 x+2\sin x.\cos^2 x+3\cos x=3\cos x.\sin^2 x$ $\Leftrightarrow \sin^3 x+2\sin x\cos^2 x+3\cos x\left(\sin^2 x+\cos^2 x\right)=3\cos x.\sin^2 x$ $\Leftrightarrow \sin^3 x+2\sin x\cos^2 x+3\cos^3 x=0 \Leftrightarrow \tan^3 x+2\tan x+3=0$ $\Leftrightarrow \tan x=-1 \vee \underbrace{\tan^2 x-\tan x+3=0}_{\Delta<0} \Leftrightarrow \tan x=\tan\left(-\frac{\pi}{4}\right) \Leftrightarrow x=-\frac{\pi}{4}+k\pi$

k)	★★	$\cos^4 x+\sin^4 x=\frac{1}{2} \Leftrightarrow \left(\cos^2 x+\sin^2 x\right)^2-2\cos^2 x\sin^2 x=\frac{1}{2} \Leftrightarrow \cos^2 x\sin^2 x=\frac{1}{4} \Leftrightarrow \frac{\sin^2 2x}{4}=\frac{1}{4}$ $\Leftrightarrow \sin^2 2x=1 \Leftrightarrow \sin 2x=1 \vee \sin 2x=-1 \Leftrightarrow 2x=\frac{\pi}{2}+2k\pi \vee 2x=-\frac{\pi}{2}+2k\pi$ $\Leftrightarrow x=\frac{\pi}{4}+k\pi \vee x=-\frac{\pi}{4}+k\pi \Leftrightarrow x=\frac{\pi}{4}+k\frac{\pi}{2}$
l)	★★⋆	$\sin x+\cos x=\frac{6}{\sec x+\csc x} \Leftrightarrow \sin x+\cos x=\frac{6}{\frac{1}{\cos x}+\frac{1}{\sin x}} \Leftrightarrow \sin x+\cos x=\frac{6\cos x\sin x}{\sin x+\cos x}$ $\Leftrightarrow \left(\sin x+\cos x\right)^2=6\cos x\sin x \Leftrightarrow 1+\sin 2x=3\sin 2x \Leftrightarrow \sin 2x=\frac{1}{2} \Leftrightarrow \sin 2x=\sin\frac{\pi}{6}$ $\Leftrightarrow 2x=\frac{\pi}{6}+2k\pi \vee 2x=\frac{5\pi}{6}+2k\pi \Leftrightarrow x=\frac{\pi}{12}+k\pi \vee x=\frac{5\pi}{12}+k\pi$
m)	★⋆	$\sqrt{3}\cos x+\sqrt{3}=\sin x \Leftrightarrow \sin x-\sqrt{3}\cos x=\sqrt{3} \Leftrightarrow \sin x-\tan\frac{\pi}{3}\cos x=\sqrt{3}$ $\Leftrightarrow \sin x\cos\frac{\pi}{3}-\sin\frac{\pi}{3}\cos x=\sqrt{3}\cos\frac{\pi}{3} \Leftrightarrow \sin\left(x-\frac{\pi}{3}\right)=\frac{\sqrt{3}}{2} \Leftrightarrow \sin\left(x-\frac{\pi}{3}\right)=\sin\frac{\pi}{3}$ $\Leftrightarrow x-\frac{\pi}{3}=\frac{\pi}{3}+2k\pi \vee x-\frac{\pi}{3}=\frac{2\pi}{3}+2k\pi \Leftrightarrow x=\frac{2\pi}{3}+2k\pi \vee x=\pi+2k\pi$
n)	★★⋆	$3\tan x=\frac{5}{\cos x}+4 \Leftrightarrow 3\sin x=5+4\cos x \Leftrightarrow \sin x-\frac{4}{3}\cos x=\frac{5}{3} \Leftrightarrow \sin x-\tan\varphi\cos x=\frac{5}{3}$ $\Leftrightarrow \sin x\cos\varphi-\sin\varphi\cos x=\frac{5}{3}\cdot\cos\varphi \overset{**}{\Leftrightarrow} \sin\left(x-\varphi\right)=1 \Leftrightarrow \sin\left(x-\varphi\right)=\sin\frac{\pi}{2}$ $\Leftrightarrow x-\varphi=\frac{\pi}{2}+2k\pi \Leftrightarrow x=\frac{\pi}{2}+\text{Bgtan}\frac{4}{3}+2k\pi \quad \left(**:\tan\varphi=\frac{4}{3} \Rightarrow \tan^2\varphi+1=\frac{25}{9} \Rightarrow \cos\varphi=\frac{3}{5}\right)$
o)	★⋆	$2\sin^2 x+\sin 2x=\cos 2x+1 \Leftrightarrow 1-\cos 2x+\sin 2x=\cos 2x+1 \Leftrightarrow \sin 2x=2\cos 2x \Leftrightarrow \tan 2x=2$ $\Leftrightarrow 2x=\text{Bgtan}2+k\pi \Leftrightarrow x=\frac{\text{Bgtan}2}{2}+k\frac{\pi}{2}$
p)	★★	$\sin^4 x+\cos^4 x=\sin x.\cos x \Leftrightarrow \left(\sin^2 x+\cos^2 x\right)^2-2\sin^2 x\cos^2 x=\sin x\cos x$ $\Leftrightarrow 1-\frac{\sin^2 2x}{2}=\frac{\sin 2x}{2} \Leftrightarrow \sin^2 2x+\sin 2x-2=0 \Leftrightarrow \sin 2x=1 \ \vee$ ~~$\sin 2x=-2$~~ $\Leftrightarrow \sin 2x=\sin\frac{\pi}{2} \Leftrightarrow 2x=\frac{\pi}{2}+2k\pi \Leftrightarrow x=\frac{\pi}{4}+k\pi$
q)	★★★⋆	$\sec x+\csc x=2\sqrt{2} \Leftrightarrow \cos x+\sin x=2\sqrt{2}\sin x\cos x \overset{!!!}{\Rightarrow} \left(\cos x+\sin x\right)^2=8\sin^2 x\cos^2 x$ $\Leftrightarrow 1+2\sin x\cos x=2\left(2\sin x\cos x\right)^2 \Leftrightarrow 1+\sin 2x=2\sin^2 2x \Leftrightarrow 2\sin^2 2x-\sin 2x-1=0$ $\Leftrightarrow \sin 2x=1 \vee \sin 2x=-\frac{1}{2} \Leftrightarrow \sin 2x=\sin\frac{\pi}{2} \vee \sin 2x=\sin\frac{-\pi}{6}$ $\Leftrightarrow 2x=\frac{\pi}{2}+2k\pi \vee 2x=-\frac{\pi}{6}+2k\pi \vee 2x=-\frac{5\pi}{6}+2k\pi$ $\Leftrightarrow x=\frac{\pi}{4}+k\pi \vee x=-\frac{\pi}{12}+k\pi \vee x=-\frac{5\pi}{12}+k\pi$  Maar, de enige oplossingen hiervan die voldoen aan de oorspronkelijke vergelijking zijn de waarden waarvoor de kwadrateringsvoorwaarde voldaan is. Dus $\cos x+\sin x$ moet hetzelfde teken hebben als $\cos x\sin x$. De hoeken waarvoor dit geldt vinden we eenvoudig op de goniometrische cirkel (aangeduid in groen). De oplossingenverzameling is dus $V=\left\{\frac{\pi}{4}+2k\pi;\frac{11\pi}{12}+2k\pi;-\frac{5\pi}{12}+2k\pi\right\}$

r) ★★

$$11\cos 2x-7\sin 2x+13=0 \Leftrightarrow \sin 2x-\frac{11}{7}\cos 2x=\frac{13}{7} \Leftrightarrow \sin 2x-\tan\varphi\cos 2x=\frac{13}{7}$$

$$\Leftrightarrow \sin 2x\cos\varphi-\sin\varphi\cos 2x=\frac{13}{7}\cos\varphi \Leftrightarrow \sin\left(2x-\varphi\right)=\frac{13}{\sqrt{170}}$$

$$\Leftrightarrow 2x-\varphi=\text{Bgsin}\frac{13}{\sqrt{170}}+2k\pi \ \vee\ 2x-\varphi=\pi-\text{Bgsin}\frac{13}{\sqrt{170}}+2k\pi$$

$$\Leftrightarrow x=\frac{\text{Bgsin}\frac{13}{\sqrt{170}}+\text{Bgtan}\frac{11}{7}}{2}+k\pi \ \vee\ x=\frac{\pi-\text{Bgsin}\frac{13}{\sqrt{170}}+\text{Bgtan}\frac{11}{7}}{2}+k\pi$$

Alternatief: stel $t=\tan x$, dan is $\sin 2x=\frac{2t}{1+t^2}$ en $\cos 2x=\frac{1-t^2}{1+t^2}$, zodat

$$11\cos 2x-7\sin 2x+13=0 \Leftrightarrow 11\frac{1-t^2}{1+t^2}-7\frac{2t}{1+t^2}+13=0 \Leftrightarrow 11\left(1-t^2\right)-14t+13\left(1+t^2\right)=0$$

$$\Leftrightarrow 2t^2-14t+24=0 \Leftrightarrow t=3\vee t=4 \Leftrightarrow \tan x=3\vee \tan x=4$$

$$\Leftrightarrow x=\text{Bgtan}3+k\pi\vee x=\text{Bgtan}4+k\pi$$

(Reken zelf eens na dat dit wel degelijk dezelfde antwoorden zijn)

s) ★★★

$$\frac{\tan x}{\tan 2x}=\frac{\tan 2x}{\tan x} \overset{x\neq\frac{k\pi}{2}}{\Leftrightarrow} \tan^2 x=\tan^2 2x \Leftrightarrow \tan x=\tan 2x\vee \tan x=\overbrace{-\tan 2x}^{=\tan(-2x)}$$

$$\Leftrightarrow x=2x+k\pi \ \vee\ x=-2x+k\pi \Leftrightarrow \cancel{x=k\pi}\vee x=k\frac{\pi}{3} \ \text{[?!]}$$

Dus x moet een veelvoud zijn van $\frac{\pi}{3}$ maar mag geen veelvoud zijn van $\frac{\pi}{2}$. De eenvoudigste manier om dit te schrijven is: $V=\left\{\frac{\pi}{3}+k\pi;\frac{2\pi}{3}+k\pi\right\}$

t) ★★★

$$2\cos 6x=2\left(\sqrt{3}+\sqrt{2}\right)\sin 3x+\sqrt{6}+2 \Leftrightarrow 2\left(1-2\sin^2 3x\right)=2\left(\sqrt{3}+\sqrt{2}\right)\sin 3x+\sqrt{6}+2$$

$$\Leftrightarrow \underbrace{4\sin^2 3x+2\left(\sqrt{3}+\sqrt{2}\right)\sin 3x+\sqrt{6}=0}_{\Delta=4\left(5+2\sqrt{6}\right)-16\sqrt{6}=\left[2\left(\sqrt{3}-\sqrt{2}\right)\right]^2} \Leftrightarrow \sin 3x=-\frac{\sqrt{2}}{2}\vee \sin 3x=\frac{\sqrt{3}}{2}$$

$$\Leftrightarrow \sin 3x=\sin\left(-\frac{\pi}{4}\right)\vee \sin 3x=\sin\left(-\frac{\pi}{3}\right)$$

$$\Leftrightarrow 3x=-\frac{\pi}{4}+2k\pi\vee 3x=-\frac{3\pi}{4}+2k\pi\vee 3x=-\frac{\pi}{3}+2k\pi\vee 3x=-\frac{2\pi}{3}+2k\pi$$

$$\Leftrightarrow x=-\frac{\pi}{12}+\frac{2}{3}k\pi\vee x=-\frac{\pi}{4}+\frac{2}{3}k\pi\vee x=-\frac{\pi}{9}+\frac{2}{3}k\pi\vee x=-\frac{2\pi}{9}+\frac{2}{3}k\pi$$

u) ★

$$2+\cos 4x+\cos 2x=0 \Leftrightarrow 2+2\cos^2 2x-1+\cos 2x=0 \Leftrightarrow \overbrace{2\cos^2 2x+\cos 2x+1=0}^{\Delta<0}$$

v) ★★*

$$\tan 6x-\tan 5x=\frac{1}{2}\sin x \Leftrightarrow \frac{\sin 6x}{\cos 6x}-\frac{\sin 5x}{\cos 5x}=\frac{1}{2}\sin x$$

$$\Leftrightarrow \sin 6x\cos 5x-\sin 5x\cos 6x=\frac{1}{2}\sin x\cos 6x\cos 5x \Leftrightarrow \sin\left(6x-5x\right)=\frac{1}{2}\sin x\cos 6x\cos 5x$$

$$\Leftrightarrow \sin x=\frac{1}{2}\sin x\cos 6x\cos 5x \Leftrightarrow \sin x\left(1-\frac{1}{2}\cos 6x\cos 5x\right)=0$$

$$\Leftrightarrow \sin x=0\vee 1-\frac{1}{2}\cos 6x\cos 5x=0 \Leftrightarrow x=k\pi\vee \cancel{\cos 6x\cos 5x=2}$$

w)	★★	$4\left(\cos^3 x-\sin^3 x\right)=5\left(\cos x-\sin x\right)$ $\Leftrightarrow 4\left(\cos x-\sin x\right)\left(\cos^2 x+\cos x\sin x+\sin^2 x\right)=5\left(\cos x-\sin x\right)$ $\Leftrightarrow \cos x-\sin x=0 \ \vee\ 4\left(1+\frac{1}{2}\sin 2x\right)=5 \Leftrightarrow \tan x=1\vee \sin 2x=\frac{1}{2}$ $\Leftrightarrow \tan x=\tan\frac{\pi}{4}\vee\sin 2x=\sin\frac{\pi}{6}\Leftrightarrow x=\frac{\pi}{4}+k\pi\vee 2x=\frac{\pi}{6}+2k\pi\vee 2x=\frac{5\pi}{6}+2k\pi$ $\Leftrightarrow x=\frac{\pi}{4}+k\pi\vee x=\frac{\pi}{12}+k\pi\vee x=\frac{5\pi}{12}+k\pi$
x)	★★*	$\frac{\sin x+1}{\cos x}=\frac{\cos x+1}{\sin x}\Leftrightarrow \sin^2 x+\sin x=\cos^2 x+\cos x\Leftrightarrow \sin^2 x-\cos^2 x=\cos x-\sin x$ $\Leftrightarrow\left(\sin x-\cos x\right)\left(\sin x+\cos x\right)=-\left(\sin x-\cos x\right)\Leftrightarrow \sin x-\cos x=0\vee\sin x+\cos x=-1$ $\Leftrightarrow \tan x=1\vee\frac{2t}{1+t^2}+\frac{1-t^2}{1+t^2}=-1\Leftrightarrow\tan x=\tan\frac{\pi}{4}\ \vee\ 1+2t-t^2=-1-t^2\Leftrightarrow x=\frac{\pi}{4}+k\pi\ \vee\ t=-1$ $\Leftrightarrow x=\frac{\pi}{4}+k\pi\ \vee\ \tan\frac{x}{2}=\tan\frac{-\pi}{4}\ \Leftrightarrow\ x=\frac{\pi}{4}+k\pi\ \vee\ \cancel{x=-\frac{\pi}{2}+2k\pi}$ Ook OPGELET als je voor de tweede vergelijking de andere methode toepast: $\sin x+\cos x=-1\Leftrightarrow\sin x+\tan\frac{\pi}{4}\cos x=-1\Leftrightarrow\sin x\cos\frac{\pi}{4}+\sin\frac{\pi}{4}\cos x=-\cos\frac{\pi}{4}$ $\Leftrightarrow\sin\left(x+\frac{\pi}{4}\right)=\sin\left(-\frac{\pi}{4}\right)\Leftrightarrow x+\frac{\pi}{4}=-\frac{\pi}{4}+2k\pi\vee x+\frac{\pi}{4}=\frac{-3\pi}{4}+2k\pi$ $\Leftrightarrow\ \cancel{x=\frac{-\pi}{2}+2k\pi}\ \vee\ \cancel{x=-\pi+2k\pi}\ \boxed{BV}$
y)	★★	$\underline{\sin x}+\sin 2x+\underline{\sin 3x}=\overline{\cos x}+\cos 2x+\overline{\cos 3x}\Leftrightarrow 2\sin 2x\cos x+\sin 2x=2\cos 2x\cos x+\cos 2x$ $\Leftrightarrow \sin 2x\left(2\cos x+1\right)=\cos 2x\left(2\cos x+1\right)\Leftrightarrow 2\cos x+1=0\vee\sin 2x=\cos 2x$ $\Leftrightarrow\cos x=-\frac{1}{2}\vee\tan 2x=1\Leftrightarrow\cos x=\cos\frac{2\pi}{3}\vee\tan 2x=\tan\frac{\pi}{4}$ $\Leftrightarrow x=\frac{2\pi}{3}+2k\pi\vee x=-\frac{2\pi}{3}+2k\pi\vee x=\frac{\pi}{8}+k\frac{\pi}{2}$
z)	★★★	$\sqrt{1+\cos x}=\sqrt{2}\sin\frac{x}{2}\overset{!!!}{\Rightarrow}1+\cos x=2\sin^2\frac{x}{2}\Leftrightarrow 1+\cos x=1-\cos x\Leftrightarrow\cos x=0\Leftrightarrow x=\frac{\pi}{2}+k\pi$ Maar wegens de kwadrateringsvoorwaarde moet $\sin\frac{x}{2}>0$, dus de oplossingen worden gegeven door: $V=\left\{\frac{\pi}{2}+\boxed{4}k\pi,\frac{3\pi}{2}+\boxed{4}k\pi\right\}$

2. Los op:

$$\sin(4\sin x)=\cos(5\cos x)\Leftrightarrow \cos(5\cos x)=\cos\left(\frac{\pi}{2}-4\sin x\right)$$

$$\Leftrightarrow 5\cos x=\frac{\pi}{2}-4\sin x+2k\pi \ \vee \ 5\cos x=-\frac{\pi}{2}+4\sin x+2k\pi$$

$$\Leftrightarrow 4\sin x+5\cos x=\frac{\pi}{2}+2k\pi \ \vee \ 4\sin x-5\cos x=\frac{\pi}{2}+2k\pi$$

We lossen de linkse vergelijking op:

Deze vergelijking is oplosbaar als en slechts als $4^2+5^2\le\left(\frac{\pi}{2}+2k\pi\right)^2 \Leftrightarrow k=0\vee k=-1$

Geval 1: $k=0$

$$4\sin x+5\cos x=\frac{\pi}{2}\overset{t=\tan\frac{x}{2}}{\Leftrightarrow}4\cdot\frac{2t}{1+t^2}+5\frac{1-t^2}{1+t^2}=\frac{\pi}{2}\Leftrightarrow(\pi+10)t^2-16t+\pi-10=0$$

$$\overset{\Delta}{\Leftrightarrow}t=\frac{8+\sqrt{164-\pi^2}}{\pi+10} \ \vee \ t=\frac{8-\sqrt{164-\pi^2}}{\pi+10}$$

$$\Leftrightarrow \ x=2Bg\tan\left(\frac{8+\sqrt{164-\pi^2}}{\pi+10}\right)+2k\pi \ \vee \ x=2Bg\tan\left(\frac{8-\sqrt{164-\pi^2}}{\pi+10}\right)+2k\pi$$

Geval 2: $k=-1$

$$4\sin x+5\cos x=\frac{-3\pi}{2}\overset{t=\tan\frac{x}{2}}{\Leftrightarrow}4\cdot\frac{2t}{1+t^2}+5\frac{1-t^2}{1+t^2}=\frac{-3\pi}{2}\Leftrightarrow(-3\pi+10)t^2-16t-3\pi-10=0$$

$$\overset{\Delta}{\Leftrightarrow}t=\frac{8+\sqrt{164-9\pi^2}}{-3\pi+10} \ \vee \ t=\frac{8-\sqrt{164-9\pi^2}}{-3\pi+10}$$

$$\Leftrightarrow \ x=2Bg\tan\left(\frac{8+\sqrt{164-9\pi^2}}{-3\pi+10}\right)+2k\pi \ \vee \ x=2Bg\tan\left(\frac{8-\sqrt{164-9\pi^2}}{-3\pi+10}\right)+2k\pi$$

De rechtse vergelijking oplossen is niet nodig... want stel je $x=\pi-x'$ in de eerste vergelijking dan krijg je de tweede vergelijking. De oplossingen van de tweede vergelijking zijn dus de supplementen van de eerste vergelijking. Samengevat krijgen we dus:

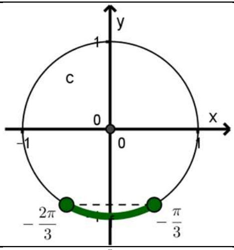
$$V=\left\{2\mathrm{Bgtan}\left(\frac{8+\sqrt{164-\pi^2}}{\pi+10}\right)+2k\pi;\ 2\mathrm{Bgtan}\left(\frac{8-\sqrt{164-\pi^2}}{\pi+10}\right)+2k\pi;\ 2\mathrm{Bgtan}\left(\frac{8+\sqrt{164-9\pi^2}}{-3\pi+10}\right)+2k\pi;\ 2\mathrm{Bgtan}\left(\frac{8-\sqrt{164-9\pi^2}}{-3\pi+10}\right)+2k\pi;\right.$$
$$\left.\pi-2\mathrm{Bgtan}\left(\frac{8+\sqrt{164-\pi^2}}{\pi+10}\right)+2k\pi;\ \pi-2\mathrm{Bgtan}\left(\frac{8-\sqrt{164-\pi^2}}{\pi+10}\right)+2k\pi;\ \pi-2\mathrm{Bgtan}\left(\frac{8+\sqrt{164-9\pi^2}}{-3\pi+10}\right)+2k\pi;\ \pi-2\mathrm{Bgtan}\left(\frac{8-\sqrt{164-9\pi^2}}{-3\pi+10}\right)+2k\pi\right\}$$

3. Los de volgende ongelijkheden op:

a) ★★

$$\sin 2x < -\frac{\sqrt{3}}{2} \Leftrightarrow -\frac{2\pi}{3} + 2k\pi < 2x < -\frac{\pi}{3} + 2k\pi$$

$$\Leftrightarrow -\frac{\pi}{3} + k\pi < x < -\frac{\pi}{6} + k\pi$$

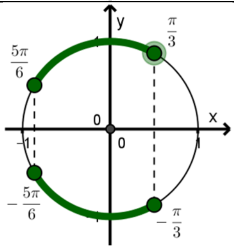


b) ★★

$$-\frac{\sqrt{3}}{2} \le \cos\frac{x}{2} \le \frac{1}{2}$$

$$\Leftrightarrow \frac{\pi}{3} + 2k\pi \le \frac{x}{2} \le \frac{5\pi}{6} + 2k\pi \;\vee\; -\frac{5\pi}{6} + 2k\pi \le \frac{x}{2} \le -\frac{\pi}{3} + 2k\pi$$

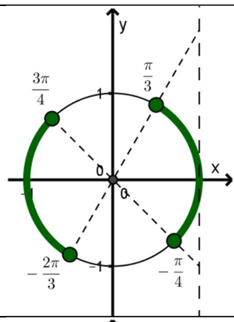
$$\Leftrightarrow \frac{2\pi}{3} + 4k\pi \le x \le \frac{5\pi}{3} + 4k\pi \;\vee\; -\frac{5\pi}{3} + 4k\pi \le x \le -\frac{2\pi}{3} + 4k\pi$$



c) ★★★

$$-1 \le \tan\left(\frac{2}{3}\pi - x\right) \le \sqrt{3} \Leftrightarrow -\frac{\pi}{4} + k\pi \le \frac{2}{3}\pi - x \le \frac{\pi}{3} + k\pi$$

$$\Leftrightarrow -\frac{\pi}{3} + k\pi \le x - \frac{2}{3}\pi \le \frac{\pi}{4} + k\pi \Leftrightarrow \frac{\pi}{3} + k\pi \le x \le \frac{11\pi}{12} + k\pi$$

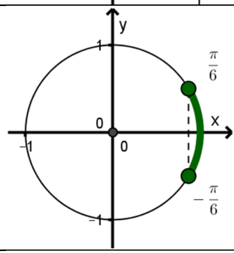


d) ★★

$$2\sin^2 2x + 3\sqrt{3}\cos 2x - 5 > 0 \Leftrightarrow 2\left(1 - \cos^2 2x\right) + 3\sqrt{3}\cos 2x - 5 > 0$$

$$\Leftrightarrow -2\cos^2 2x + 3\sqrt{3}\cos 2x - 3 > 0 \Leftrightarrow \frac{\sqrt{3}}{2} < \cos 2x < \sqrt{3}$$

$$\Leftrightarrow -\frac{\pi}{6} + 2k\pi < 2x < \frac{\pi}{6} + 2k\pi \Leftrightarrow \Leftrightarrow -\frac{\pi}{12} + k\pi < x < \frac{\pi}{12} + k\pi$$

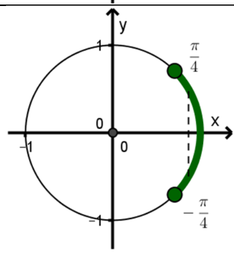


e) ★★

$$\tan x . \tan\frac{x}{2} \ge 0 \overset{t=\tan\frac{x}{2}}{\Leftrightarrow} \frac{2t}{1-t^2}\cdot t \ge 0 \Leftrightarrow \frac{2t^2}{1-t^2} \ge 0$$

$$\Leftrightarrow -1 < \mathrm{t} < 1 \Leftrightarrow -1 < \tan\frac{x}{2} < 1 \Leftrightarrow -\frac{\pi}{4} + k\pi < \frac{x}{2} < \frac{\pi}{4} + k\pi$$

$$\Leftrightarrow -\frac{\pi}{2} + 2k\pi < x < \frac{\pi}{2} + 2k\pi$$



f) ★★★

$$\sin 2x < \cos x \Leftrightarrow 2\sin x\cos x - \cos x < 0 \Leftrightarrow \cos x\left(2\sin x - 1\right) < 0$$

De functie in het linkerlid heeft periode $P = 2\pi$.

We bekijken het tekenverloop binnen één periode $\left[0, 2\pi\right]$:

x	0	$\pi/6$		$\pi/2$		$5\pi/6$		$3\pi/2$	2π
$2\sin x - 1$	-	0	+	+	+	0	-	-	-
$\cos x$	+	+	+	0	-	-	-	0	+
LL	-	0	+	0	-	0	+	0	-

Dus: $-\frac{\pi}{2} + 2k\pi < x < \frac{\pi}{6} + 2k\pi \;\vee\; \frac{\pi}{2} + 2k\pi < x < \frac{5\pi}{6} + 2k\pi$