1. Bereken de volgende onbepaalde integralen (een substitutie is aangewezen):

a) $\int \sin(3x-4)\,dx = \frac{1}{3}\int \sin(3x-4)\,d(3x-4) = -\frac{1}{3}\cos(3x-4)+C$

b) $\int e^{2x-3}dx = \frac{1}{2}\int e^{2x-3}d(2x-3) = \frac{1}{2}e^{2x-3}+C$

c) $\int \tan x\,dx = \int \frac{\sin x}{\cos x}dx = -\int \frac{d(\cos x)}{\cos x} = -\ln|\cos x|+C$

d) $\int \frac{1}{\sqrt[5]{4x-7}}dx = \frac{1}{4}\int (4x-7)^{-\frac{1}{5}}\,d(4x-7) = \frac{1}{4}\frac{(4x-7)^{\frac{4}{5}}}{4/5}+C = \frac{5}{16}\sqrt[5]{(4x-7)^4}+C$

e) $\int x(4x^2-2)^7\,dx = \frac{1}{8}\int t^7 dt = \frac{1}{8}\cdot\frac{t^8}{8}+C = \frac{(4x^2-2)^8}{64}+C$

Stel $4x^2-2=t \Rightarrow 8xdx=dt \Rightarrow xdx=\frac{dt}{8}$

f) $\int \frac{\ln x}{x}dx = \int t\,dt = \frac{t^2}{2}+C = \frac{(\ln x)^2}{2}+C$

Stel $\ln x = t \Rightarrow \frac{dx}{x}=dt$

g) $\int \frac{dx}{x\ln x} = \int \frac{dt}{t} = \ln|t|+C = \ln|\ln(x)|+C$

Stel $\ln x = t \Rightarrow \frac{dx}{x}=dt$

h) $\int \frac{3x^2-1}{\sqrt[4]{x^3-x+3}}dx = \int \frac{dt}{\sqrt[4]{t}} = \int t^{-\frac{1}{4}}dt = \frac{t^{\frac{3}{4}}}{3/4}+C = \frac{4}{3}\sqrt[4]{(x^3-x+3)^3}+C$

Stel $x^3-x+1=t \Rightarrow (3x^2-1)dx=dt$

i) $\int \sin^4 x\cos x\,dx = \int t^4\,dt = \frac{t^5}{5}+C = \frac{\sin^5 x}{5}+C$

Stel $\sin x = t \Rightarrow \cos x\,dx = dt$

j) $\int \frac{x^3}{\sin^2(x^4)}dx = \frac{1}{4}\int \frac{dt}{\sin^2 t} = -\frac{\cot t}{4}+C = -\frac{\cot(x^4)}{4}+C$

Stel $x^4=t \Rightarrow 4x^3dx=dt \Rightarrow x^3dx=\frac{dt}{4}$

k) $\int \frac{x-3}{x^2+1}dx = \int \frac{x}{x^2+1}dx - 3\int \frac{dx}{x^2+1} = \frac{1}{2}\ln(x^2+1) - 3\,\mathrm{Bgtan}\,x + C$

Stel $x^2+1=t \Rightarrow 2x\,dx=dt$. Dan is $\int \frac{x}{x^2+1}dx = \frac{1}{2}\int\frac{dt}{t} = \frac{1}{2}\ln|t|+C = \frac{1}{2}\ln(x^2+1)+C$

l) $\displaystyle\int \frac{4x}{\sqrt{1-x^2}}\,dx = -2\int \frac{dt}{\sqrt{t}} = -2\int t^{-\frac{1}{2}}dt = -2\frac{t^{\frac{1}{2}}}{1/2} + C = -4\sqrt{t} + C = -4\sqrt{1-x^2} + C$

Stel $1-x^2 = t \Rightarrow -2x\,dx = dt$

m) $\displaystyle\int \frac{4x}{\sqrt{1-x^4}}\,dx = 2\int \frac{dt}{\sqrt{1-t^2}} = 2\,\text{Bgsin}\,t + C = 2\,\text{Bgsin}\,x^2 + C$

Stel $x^2 = t \Rightarrow 2x\,dx = dt$

n) $\displaystyle\int \cos^3 x\,dx = \int (1-\sin^2 x)\cos x\,dx = \int \cos x\,dx - \int \sin^2 x \cos x\,dx = \sin x - \frac{\sin^3 x}{3} + C$

Stel $\sin x = t \Rightarrow \cos x\,dx = dt$. Dan is $\displaystyle\int \sin^2 x\cos x\,dx = \int t^2 dt = \frac{t^3}{3} + C = \frac{\sin^3 x}{3} + C$

o) $\displaystyle\int \frac{\text{Bgsin}\,x}{\sqrt{1-x^2}}\,dx = \int t\,dt = \frac{t^2}{2} + C = \frac{(\text{Bgsin}\,x)^2}{2} + C$

Stel $\displaystyle \text{Bgsin}\,x = t \Rightarrow \frac{dx}{\sqrt{1-x^2}} = dt$

p) $\displaystyle\int \frac{\tan\left(\ln\left(\sqrt{x}\right)\right)}{x}\,dx = 2\int \tan t\,dt \overset{\text{oef c}}{=} -2\ln|\cos t| + C = -2\ln\left|\cos\left(\ln\left(\sqrt{x}\right)\right)\right| + C$

Stel $\displaystyle \ln\left(\sqrt{x}\right) = \frac{1}{2}\ln x = t \Rightarrow \frac{1}{2}\cdot\frac{dx}{x} = dt$

q) $\displaystyle\int \frac{\cos 2x}{\sin^2 x\cos^2 x}\,dx = 4\int \frac{\cos 2x}{\sin^2 2x}\,dx = 2\int \frac{dt}{t^2} = -\frac{2}{t} + C = -\frac{2}{\sin 2x} + C$

Stel $\sin 2x = t \Rightarrow 2\cos 2x\,dx = dt$ (merk ook op dat $\sin 2x = 2\sin x\cos x \Rightarrow \sin^2 2x = 4\sin^2 x\cos^2 x$)

r) $\displaystyle\int \frac{x\cdot e^{\sqrt{1-x^2}}}{\sqrt{1-x^2}}\,dx = -\int e^t dt = -e^t + C = -e^{\sqrt{1-x^2}} + C$

Stel $\displaystyle \sqrt{1-x^2} = t \Rightarrow \frac{-\cancel{2}x}{\cancel{2}\sqrt{1-x^2}}\,dx = dt$

s) $\displaystyle\int \frac{\sin 2x}{(2+\sin x)^2}\,dx = \int \frac{2\sin x\cos x}{(2+\sin x)^2}\,dx = \int \frac{2(t-2)}{t^2}\,dt = 2\int\frac{dt}{t} - 4\int\frac{dt}{t^2} = 2\ln|t| + \frac{4}{t} + C = \ldots$

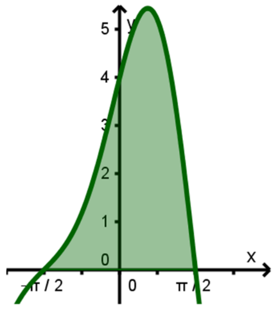
Stel $2+\sin x = t \Rightarrow \cos x\,dx = dt$ (en $\sin x = t-2$)

$\displaystyle \ldots = 2\ln(2+\sin x) + \frac{4}{2+\sin x} + C$

t) $\displaystyle\int \frac{dx}{e^x + e^{-x}} = \int \frac{dx}{e^x+e^{-x}}\cdot\frac{e^x}{e^x} = \int \frac{e^x}{(e^x)^2+1}\,dx = \int \frac{dt}{t^2+1} = \text{Bgtan}(e^x) + C$

Stel $e^x = t \Rightarrow e^x dx = dt$

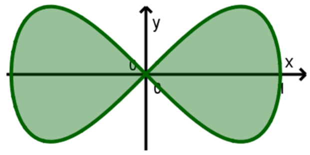
2. Op de grafiek zie je dat de functie $f(x)=(2+\sin x)^2\cdot\cos x$ positief is in het interval $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$. Bereken in dat interval de oppervlakte onder de kromme.



$$S=\int_{-\pi/2}^{\pi/2}(2+\sin x)^2\cdot\cos x\,dx=\int_1^3 t^2dt=\frac{1}{3}\left[t^3\right]_1^3=\frac{1}{3}\left(3^3-1^3\right)=\frac{26}{3}$$

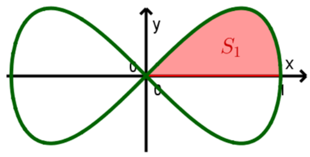
Stel $2+\sin x=t\Rightarrow\cos x\,dx=dt$ $\boxed{x=\pi/2\Rightarrow t=3\ \ en\ \ x=-\pi/2\Rightarrow t=1}$

3. Je ziet de grafiek getekend van de kromme $\mathcal{K}\leftrightarrow y^2=x^2-x^4$.



Bereken de oppervlakte van het gebied dat deze kromme omsluit.

De kromme bestaat uit twee functies: $y=x\sqrt{1-x^2}$ en $y=-x\sqrt{1-x^2}$.



De oppervlakte kan berekend worden door vier keer de in rood aangeduide oppervlakte S_1 te berekenen. We vinden:

$$S=4S_1=4\int_0^1 x\sqrt{1-x^2}\,dx=-2\int_1^0\sqrt{t}=-2\left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^0=-\frac{4}{3}\left[t\sqrt{t}\right]_1^0=-\frac{4}{3}(0-1)=\frac{4}{3}$$

Stel $1-x^2=t\Rightarrow-2x\,dx=dt$ $\boxed{x=1\Rightarrow t=0\ \ en\ \ x=0\Rightarrow t=1}$

4. Bereken de volgende onbepaalde integralen (partiële integratie is aangewezen):

a) $\int\text{Bgsin}^2 2x\,dx\overset{\boxed{1}}{=}x\cdot\text{Bgsin}^2 2x-4\int\frac{x\cdot\text{Bgsin}\,2x}{\sqrt{1-4x^2}}dx\overset{\boxed{2}}{=}x\cdot\text{Bgsin}^2 2x+\text{Bgsin}\,2x\cdot\sqrt{1-4x^2}-\int 2\,dx=\ldots$

1) $u=\text{Bgsin}^2 2x\Rightarrow du=\frac{4\text{Bgsin}\,2x}{\sqrt{1-4x^2}}dx$ en $dv=dx\Rightarrow v=x$

2) $u=\text{Bgsin}\,2x\ \Rightarrow\ du=\frac{2}{\sqrt{1-4x^2}}dx$ en $dv=\frac{x\,dx}{\sqrt{1-4x^2}}\Rightarrow v=\int\frac{x\,dx}{\sqrt{1-4x^2}}\overset{\boxed{*}}{=}-\frac{1}{4}\sqrt{1-4x^2}$

*: $1-4x^2=t\Rightarrow-8x\,dx=dt$, zodat $\int\frac{x\,dx}{\sqrt{1-4x^2}}=-\frac{1}{8}\int\frac{dt}{\sqrt{t}}=-\frac{1}{8}\cdot\frac{\sqrt{t}}{1/2}=-\frac{1}{4}\sqrt{1-4x^2}$

$\ldots=x\cdot\text{Bgsin}^2 2x+\text{Bgsin}\,2x\cdot\sqrt{1-4x^2}-2x+C$

b) $\int\left(x^2-x+1\right)\sin x\,dx\overset{\boxed{1}}{=}-\left(x^2-x+1\right)\cos x+\int(2x-1)\cos x\,dx=\ldots$

1) $u=x^2-x+1\Rightarrow du=(2x-1)dx$ en $dv=\sin x\,dx\Rightarrow v=-\cos x$

$\ldots\overset{\boxed{2}}{=}-\left(x^2-x+1\right)\cos x+(2x-1)\sin x-\int 2\sin x\,dx=-\left(x^2-x+1\right)\cos x+(2x-1)\sin x+2\cos x+C=\ldots$

2) $u=2x-1\Rightarrow du=2dx$ en $dv=\cos x\,dx\Rightarrow v=\sin x$

$\ldots=\left(-x^2+x+1\right)\cos x+(2x-1)\sin x+C$

c) $\int x \cdot \text{Bgtan}\, x\, dx \overset{\boxed{1}}{=} \frac{x^2}{2} \cdot \text{Bgtan}\, x - \frac{1}{2} \int \frac{x^2}{1+x^2} dx \overset{\boxed{2}}{=} \frac{x^2}{2} \cdot \text{Bgtan}\, x + \frac{1}{2} \cdot \text{Bgtan}\, x - \frac{x}{2} + C$

1) $u = \text{Bgtan}\, x \;\Rightarrow\; du = \frac{dx}{1+x^2}$ en $dv = x\, dx \;\Rightarrow\; v = \frac{x^2}{2}$

2) $\int \frac{x^2}{1+x^2} dx = \int \frac{1+x^2-1}{1+x^2} dx = \int dx - \int \frac{dx}{1+x^2} = x - \text{Bgtan}\, x + C$

d) $I = \int e^{-x} \sin x\, dx \overset{\boxed{1}}{=} -e^{-x} \cos x - \int e^{-x} \cos x\, dx \overset{\boxed{2}}{=} -e^{-x} \cos x - e^{-x} \sin x - \int e^{-x} \sin x\, dx$

1) $u = e^{-x} \;\Rightarrow\; du = -e^{-x} dx$ en $dv = \sin x\, dx \;\Rightarrow\; v = -\cos x$

2) $u = e^{-x} \;\Rightarrow\; du = -e^{-x} dx$ en $dv = \cos x\, dx \;\Rightarrow\; v = \sin x$

$\Leftrightarrow 2I = -e^{-x} \cos x - e^{-x} \sin x + C \Leftrightarrow I = -\frac{1}{2} e^{-x} \cos x - \frac{1}{2} e^{-x} \sin x + C$

e) $\int \ln\left(4+x^2\right) dx \overset{\boxed{1}}{=} x \cdot \ln\left(4+x^2\right) - 2\int \frac{x^2}{4+x^2} dx \overset{\boxed{2}}{=} x \cdot \ln\left(4+x^2\right) - 2x + 4\text{Bgtan}\, \frac{x}{2} + C$

1) $u = \ln\left(4+x^2\right) \;\Rightarrow\; du = \frac{2x\, dx}{4+x^2}$ en $dv = dx \;\Rightarrow\; v = x$

2) $\int \frac{x^2}{4+x^2} dx = \int \frac{4+x^2-4}{4+x^2} dx = \int dx - 4\int \frac{dx}{4+x^2} = x - 2\text{Bgtan}\, \frac{x}{2} + C$

f) $\int e^{\sin x} \cos^3 x\, dx = \int e^{\sin x} \left(1-\sin^2 x\right) \cos x\, dx \overset{\boxed{1}}{=} \int \left(1-t^2\right) e^t dt \overset{\boxed{2}}{=} \left(1-t^2\right) e^t + 2\int t e^t dt = \ldots$

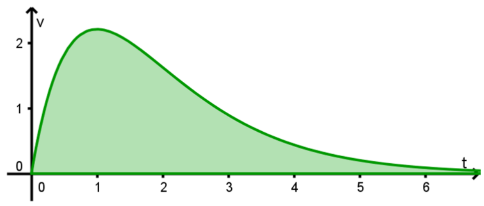
1) stel $t = \sin x \Rightarrow dt = \cos x\, dx$

3) $u = t \;\Rightarrow\; du = dt$ en $dv = e^t dt \;\Rightarrow\; v = e^t$

2) $u = 1-t^2 \;\Rightarrow\; du = -2t\, dt$ en $dv = e^t dt \;\Rightarrow\; v = e^t$

$\ldots \overset{\boxed{3}}{=} \left(1-t^2\right) e^t + 2te^t - 2\int e^t dt + C = \left(1-t^2\right) e^t + 2te^t - 2e^t + C = e^{\sin x} \left(-\sin^2 x + 2\sin x - 1\right) + C$

5. Een fiets rijdt lek op tijdstip $t = 0$. De snelheid waarmee de fietsband lucht verliest wordt gegeven door de functie $v(t) = 6te^{-t}$, met t in minuten en v in liter per minuut.



a) ★★ Hoeveel lucht is de band al verloren na 3 minuten?

$L = \int_0^3 v(t)\, dt = \int_0^3 6te^{-t} dt \overset{\otimes}{=} -6\left[e^{-t}(t+1)\right]_0^3 = -24e^{-3} + 6 \approx 4{,}80511$ (dus ongeveer 4,80 liter lucht)

$\otimes$: $\int 6te^{-t} dt = -6te^{-t} + 6\int e^{-t} dt = -6te^{-t} - 6e^{-t} + C = -6e^{-t}(t+1) + C$

b) ★★★ Toon aan dat er in totaal 6 liter lucht in de band zit.

$L_\infty = \int_0^{+\infty} v(t)\, dt = \lim_{k\to+\infty} \int_0^k 6te^{-t} dt \overset{\otimes}{=} \lim_{k\to+\infty} \left(-6\left[e^{-t}(t+1)\right]_0^k\right) = \lim_{k\to+\infty} \left(-6e^{-k}(k+1) + 6\right) \overset{\boxed{\Xi}}{=} 0 + 6 = 6 \quad \square$

$\boxed{\Xi}$: $\lim_{k\to+\infty} \left(e^{-k}(k+1)\right) = \lim_{k\to+\infty} \frac{k+1}{e^k} \overset{\frac{+\infty}{+\infty}}{\underset{\boxed{H}}{=}} \lim_{k\to+\infty} \frac{1}{e^k} = \frac{1}{+\infty} = 0$

6. Bereken de volgende onbepaalde integralen:

a) $$\int \frac{x}{9x^2+6x+5}dx = \int \frac{\frac{1}{18}(18x+6)-\frac{1}{3}}{9x^2+6x+5}dx = \frac{1}{18}\int \frac{d(9x^2+6x+5)}{9x^2+6x+5} - \frac{1}{9}\int \frac{d(3x+1)}{(3x+1)^2+4}$$

$$= \frac{1}{18}\ln\left|9x^2+6x+5\right| - \frac{1}{18}\text{Bgtan}\frac{3x+1}{2}+C$$

b) $$\int \frac{2x+1}{\sqrt{9+16x-4x^2}}dx = \int \frac{-\frac{1}{4}(16-8x)+5}{\sqrt{9+16x-4x^2}}dx = -\frac{1}{4}\int \frac{d(9+16x-4x^2)}{\sqrt{9+16x-4x^2}} + \frac{5}{2}\int \frac{d(2x-4)}{\sqrt{25-(2x-4)^2}}$$

$$= -\frac{1}{2}\sqrt{9+16x-4x^2} + \frac{5}{2}\text{Bgsin}\frac{2x-4}{5}+C$$

c) $$\int \frac{3x+2}{x^3+x^2-2}dx \overset{SIP}{=} \int \frac{1}{x-1}dx - \int \frac{x}{x^2+2x+2}dx \overset{*}{=} \ln|x-1| - \frac{1}{2}\ln\left|x^2+2x+2\right| + \text{Bgtan}(x+1)+C$$

$$\frac{3x+2}{x^3+x^2-2} = \frac{A}{x-1}+\frac{Bx+C}{x^2+2x+2} = \frac{A(x^2+2x+2)+(Bx+C)(x-1)}{(x-1)(x^2+2x+2)} = \frac{(A+B)x^2+(2A-B+C)x+2A-C}{(x-1)(x^2+2x+2)}$$

$$\Leftrightarrow \begin{cases} A+B=0 \\ 2A-B+C=3 \\ 2A-C=2 \end{cases} \Leftrightarrow \begin{cases} A=1 \\ B=-1 \\ C=0 \end{cases} \Rightarrow \boxed{\frac{3x+2}{x^3+x^2-2} = \frac{1}{x-1} - \frac{x}{x^2+2x+2}}$$

$$\int \frac{x}{x^2+2x+2}dx = \int \frac{\frac{1}{2}(2x+2)-1}{x^2+2x+2}dx = \frac{1}{2}\int \frac{d(x^2+2x+2)}{x^2+2x+2} - \int \frac{d(x+1)}{(x+1)^2+1} = \frac{1}{2}\ln\left|x^2+2x+2\right| - \text{Bgtan}(x+1)$$

d) $$\int \frac{dx}{x^3+8} = \frac{1}{12}\int \frac{dx}{x+2} + \frac{1}{12}\int \frac{-x+4}{x^2-2x+4}dx = \frac{1}{12}\ln|x+2| - \frac{1}{24}\ln\left|x^2-2x+4\right| + \frac{\sqrt{3}}{12}\text{ Bgtan}\frac{x-1}{\sqrt{3}}+C$$

$$\frac{1}{x^3+8} = \frac{A}{x+2}+\frac{Bx+C}{x^2-2x+4} = \frac{A(x^2-2x+4)+(Bx+C)(x+2)}{(x+2)(x^2-2x+4)} = \frac{(A+B)x^2+(-2A+2B+C)x+4A+2C}{x^3+8}$$

$$\Leftrightarrow \begin{cases} A+B=0 \\ -2A+2B+C=0 \\ 4A+2C=1 \end{cases} \Leftrightarrow \begin{cases} A=1/12 \\ B=-1/12 \\ C=1/3 \end{cases} \Rightarrow \boxed{\frac{1}{x^3+8} = \frac{1}{12}\cdot\frac{1}{x+2} + \frac{1}{12}\cdot\frac{-x+4}{x^2-2x+4}}$$

$$\int \frac{-x+4}{x^2-2x+4}dx = \int \frac{-\frac{1}{2}(2x-2)+3}{x^2-2x+4}dx = -\frac{1}{2}\int \frac{d(x^2-2x+4)}{x^2-2x+4} + 3\int \frac{d(x-1)}{(x-1)^2+3} = -\frac{1}{2}\ln\left|x^2-2x+4\right| + \sqrt{3}\text{ Bgtan}\frac{x-1}{\sqrt{3}}+C$$

e) $$\int \frac{\sqrt{x}}{1+x}dx = \int \frac{t}{1+t^2}2t\,dt = \int \frac{2t^2}{1+t^2}dt = \int \frac{2(1+t^2)-2}{1+t^2}dt = \int 2dt - 2\int \frac{dt}{1+t^2} = 2t - 2\text{ Bgtan } t + C$$

$$= 2\sqrt{x} - 2\text{ Bgtan}\sqrt{x}+C$$ Stel $x = t^2 \Rightarrow dx = 2t\,dt$

f) $$\int \cos^2 8x\,dx \overset{Carnot}{=} \int \frac{1+\cos 16x}{2}dx = \frac{x}{2}+\frac{1}{32}\sin 16x + C$$

g) $$\int \tan^3 3x\,dx = \int \tan^2 3x \cdot \tan 3x\,dx = \int (\sec^2 3x - 1)\cdot \tan 3x\,dx = \frac{1}{3}\int \tan 3x\, d(\tan 3x) - \int \tan 3x\,dx$$

$$= \frac{1}{6}\tan^2 3x + \frac{1}{3}\ln\left|\cos 3x\right| + C$$

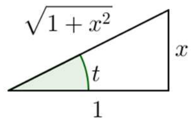
h) $\displaystyle\int \frac{\cot x + \csc x}{\sin x}\,dx = \int \frac{\cos x}{\sin^2 x}\,dx + \int \frac{dx}{\sin^2 x} = -\frac{1}{\sin x} - \cot x + C$

i) $\displaystyle\int \sec^4 x\,dx = \int \sec^2 x \cdot \sec^2 x\,dx = \int \left(1+\tan^2 x\right) d\left(\tan x\right) = \tan x + \frac{1}{3}\tan^3 x + C$

j) $\displaystyle\int \frac{\sqrt{x^2-25}}{x}\,dx = \int \frac{5\tan t \cdot \cancel{5\sec t} \cdot \tan t}{\cancel{5\sec t}}\,dt = 5\int \tan^2 t\,dt = 5\int \frac{1-\cos^2 t}{\cos^2 t}\,dt = 5\tan t - 5t + C$

$\displaystyle = \sqrt{x^2-25} - 5\,\text{Bgsec}\frac{x}{5} + C$ Stel $x = 5\sec t \Rightarrow dx = 5\sec t \tan t\,dt$ en $\sqrt{x^2-25} = 5\tan t$ $(x>5)$

k) $\displaystyle\int \frac{dx}{x\sqrt{1+x^2}} = \int \frac{\sec^2 t\,dt}{\tan t \sec t} = \int \csc t\,dt = -\ln\left|\csc t + \cot t\right| + C = -\ln\left|\frac{\sqrt{1+x^2}}{x} + \frac{1}{x}\right| + C$



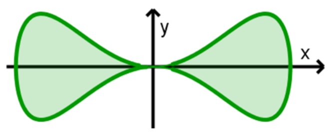
Kan eventueel geschreven worden als: $\displaystyle -\ln\left|\frac{1+\sqrt{1+x^2}}{x}\right| + C = \ln\left|\frac{x}{1+\sqrt{1+x^2}}\right| + C$

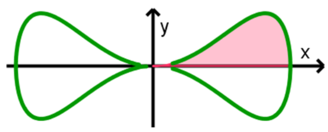
Stel $x = \tan t \Rightarrow dx = \sec^2 t\,dt$ en $\sqrt{1+x^2} = \sec t$

l) $\displaystyle\int \frac{x^3}{\sqrt{4-x^2}}\,dx = \int \frac{8\sin^3 t}{\cancel{2\cos t}}\,\cancel{2\cos t}\,dt = -8\int \left(1-\cos^2 t\right) d\cos t = -8\cos t + \frac{8}{3}\cos^3 t = -4\sqrt{4-x^2} + \frac{1}{3}\sqrt{\left(4-x^2\right)^3} + C$

Stel $x = 2\sin t \Rightarrow dx = 2\cos t\,dt$ en $\sqrt{4-x^2} = 2\cos t$

7. Je ziet de grafiek getekend van de kromme $\mathcal{K} \leftrightarrow y^2 = x^4 - x^6$.





Bereken de oppervlakte van het gebied dat deze kromme omsluit.

De kromme bestaat uit twee functies: $y = x^2\sqrt{1-x^2}$ en $y = -x^2\sqrt{1-x^2}$.

Deze functies hebben als domein $\left[-1,1\right]$.

De oppervlakte kan berekend worden door vier keer de in rood aangeduide oppervlakte S_1 te berekenen.

$$S = 4\cdot S_1 = 4\int_0^1 x^2\sqrt{1-x^2}\,dx \overset{*}{=} 4\int_0^{\pi/2} \sin^2 t \cos t \cos t\,dt = 4\int_0^{\pi/2} \sin^2 t \cos^2 t\,dt = \int_0^{\pi/2} \sin^2 2t\,dt = \int_0^{\pi/2} \frac{1-\cos 4t}{2}\,dt$$

$$= \int_0^{\pi/2} \frac{1-\cos 4t}{2}\,dt = \left[\frac{t}{2} - \frac{\sin 4t}{8}\right]_0^{\pi/2} = \frac{\pi}{4}$$

Stel $x = \sin t \Rightarrow dx = \cos t\,dt$ en $\sqrt{1-x^2} = \cos t$ (met $0 \le t \le \frac{\pi}{2}$). Als $x = 0$ is $t = 0$ en als $x = 1$ in $t = \frac{\pi}{2}$.